

READ AND FOLLOW ALL DIRECTIONS. CIRCLE YOUR FINAL ANSWERS.
SHOW ALL WORK TO RECEIVE FULL CREDIT. NO CALCULATORS.

1. (10 points) Let $f(x) = -3x^2 + 12x - 9$

(a) Circle the correct option:

The graph of f (opens up / opens down) and has a highest / lowest) point.

(b) Find the vertex of the graph of $f(x)$.

$$h = \frac{-b}{2a} = \frac{-12}{2 \cdot (-3)} = \frac{-12}{-6} = 2$$

$$k = f(h) = f(2) = -3(2)^2 + 12 \cdot 2 - 9 \\ = -3 \cdot 4 + 24 - 9 = -12 + 15 = 3$$

Vertex $(h, k) = (2, 3)$

(c) Find the x-intercept(s) of $f(x)$.

x-intercepts are where $f(x) = 0$

$$-3(x-2)^2 + 3 = 0$$

$$-3(x-2)^2 = -3$$

$$(x-2)^2 = 1$$

$$x-2 = \pm 1 \quad \text{so } x = 2 \pm 1 = \text{3 or 1}$$

(d) Find the y-intercept of $f(x)$

y-intercept is $f(0) = -3(0)^2 + 12(0) - 9 = -9$

Quiz #5

2. (6 points) Let $f(x) = 2x^2 - 12x + 18$

(a) Find the vertex of the graph of $f(x)$.

$$h = \frac{-b}{2a} = \frac{-(-12)}{2 \cdot 2} = \frac{12}{4} = 3$$

$$k = f(h) = f(3) = 2 \cdot 3^2 - 12 \cdot 3 + 18 \\ = 2 \cdot 9 - 36 + 18 = 0$$

$$\text{vertex } (h, k) = (3, 0)$$

(b) What is the axis of symmetry of the graph of $f(x)$?

The axis of symmetry is the vertical line through the vertex $x = 3$

3. (4 points) Determine the quadratic function with vertex $(1, -3)$ which passes through the point $(3, 5)$. (Your answer should be of the form $f(x) = a(x - h)^2 + k$ where you supply a , h , and k).

$$f(x) = a(x - 1)^2 - 3 \quad \text{since vertex is } (1, -3)$$

$$5 = f(3) = a(3 - 1)^2 - 3 \quad \text{since } (3, 5) \text{ on the graph}$$

$$8 = a(2)^2$$

$$8 = 4a$$

$$a = 2$$

$$f(x) = 2(x - 1)^2 - 3$$

4. (2 points) Can a quadratic function have a range of $(-\infty, \infty)$? Justify your answer.

No. Every quadratic function attains a maximum or minimum at its vertex, so its range excludes all values larger than the max (or alternately, smaller than the min)