

READ AND FOLLOW ALL DIRECTIONS. CIRCLE YOUR FINAL ANSWERS.
SHOW ALL WORK TO RECEIVE FULL CREDIT. NO CALCULATORS.

1. (4 points) Let $f(x) = \frac{1}{x-2}$ and $g(x) = \frac{2x^2}{x+4}$
(a) Find $(f \circ g)(4)$.

$$(f \circ g)(4) = f(g(4)) = f\left(\frac{2 \cdot 4^2}{4+4}\right) = f\left(\frac{2 \cdot 16}{2 \cdot 4}\right) = f(4) = \frac{1}{4-2} = \frac{1}{2}$$

- (b) Find $(g \circ f)(3)$.

$$(g \circ f)(3) = g(f(3)) = g\left(\frac{1}{3-2}\right) = g(1) = \frac{2 \cdot 1^2}{1+4} = \frac{2}{5}$$

2. (4 points) Verify that the functions $f(x) = 3x + 4$ and $g(x) = \frac{1}{3}(x - 4)$ are inverses of each other by showing that $f \circ g(x) = x$ and $g \circ f(x) = x$.

$$\begin{aligned} f(g(x)) &= 3\left(\frac{1}{3}(x-4)\right) + 4 \\ &= x - 4 + 4 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= \frac{1}{3}\left((3x+4)-4\right) \\ &= \frac{1}{3}(3x) = x \end{aligned}$$

So we conclude $g(x) = f^{-1}(x)$

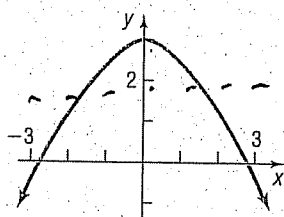
Quiz #8

3. (4 points) Determine whether or not each of the given functions is one-to-one. Explain your reasoning.

(a) $\{(-2, 6); (-1, 3); (0, 2); (1, 5); (2, 8)\}$ one-to-one

Each input gives a distinct output (i.e. no two inputs have the same output)

(b)



Not one-to-one

The graph of this function fails the horizontal line test

4. (8 points) The function $f(x) = \frac{2x}{x-6}$ is one-to-one. Find its inverse function $f^{-1}(x)$. For extra credit, verify $f^{-1}(f(x)) = x$.

$$x = f(f^{-1}(x)) = \frac{2f^{-1}(x)}{f^{-1}(x) - 6}$$

$$x(f^{-1}(x) - 6) = 2f^{-1}(x)$$

$$\begin{array}{r} x f^{-1}(x) - 6x = 2 f^{-1}(x) \\ +6x \qquad \qquad +6x \\ -2 f^{-1}(x) \qquad -2 f^{-1}(x) \end{array}$$

$$x f^{-1}(x) - 2 f^{-1}(x) = 6x$$

$$f^{-1}(x) \cdot (x - 2) = 6x$$

$$\boxed{f^{-1}(x) = \frac{6x}{x-2}}$$

XC

$$\begin{aligned} f^{-1}(f(x)) &= \frac{6\left(\frac{2x}{x-6}\right)}{\frac{2x}{x-6} - 2\left(\frac{x-6}{x-6}\right)} \\ &= \frac{12x}{x-6} \\ &= \frac{2x - 2x + 12}{x-6} \\ &= \frac{12x}{x-6} \cdot \frac{x-6}{12} \\ &= \frac{12x}{12} = x \end{aligned}$$