

READ AND FOLLOW ALL DIRECTIONS. CIRCLE YOUR FINAL ANSWERS.
SHOW ALL WORK TO RECEIVE FULL CREDIT. NO CALCULATORS.

1. (4 points) Let $f(x) = \frac{3}{x-1}$ and $g(x) = \frac{2}{x}$

(a) Find $(f \circ g)(1)$.

$$f \circ g(1) = f(g(1)) = f\left(\frac{2}{1}\right) = f(2) = \frac{3}{2-1} = \frac{3}{1} = 3$$

(b) Find $(g \circ f)(4)$.

$$g \circ f(4) = g(f(4)) = g\left(\frac{3}{4-1}\right) = g(1) = \frac{2}{1} = 2$$

2. (6 points) The function $f(x) = \frac{3}{x+2}$ is one-to-one. Find its inverse function $f^{-1}(x)$.

$$x = f(f^{-1}(x)) = \frac{3}{f^{-1}(x)+2}$$

$$x(f^{-1}(x)+2) = 3$$

$$xf^{-1}(x) + 2x = 3$$

$$xf^{-1}(x) = 3 - 2x$$

$$f^{-1}(x) = \frac{3-2x}{x}$$

Quiz #9

3. (4 points) Solve for x : $(\sqrt[3]{2})^{3x} = 4^2 \cdot 2 \cdot \frac{2^{24}}{2^{12}}$

$$\begin{aligned} (2^{1/3})^{3x} &= (2^2)^2 \cdot 2 \cdot 2^{24-12} \\ 2^{1/3 \cdot 3x} &= 2^{2 \cdot 2} \cdot 2 \cdot 2^{12} \\ 2^x &= 2^4 \cdot 2 \cdot 2^{12} = 2^{4+1+12} = 2^{17} \end{aligned}$$

$$\boxed{x = 17}$$

4. (6 points) Solve for x : $9^{2x} \cdot 27^{x^2} = 3^{-1}$

$$(3^2)^{2x} \cdot (3^3)^{x^2} = 3^{-1}$$

$$3^{4x} \cdot 3^{3x^2} = 3^{-1}$$

$$3^{3x^2+4x} = 3^{-1}$$

$$3x^2+4x = -1$$

$$3x^2+4x+1 = 0$$

$$(3x+1)(x+1) = 0$$

$$\boxed{x = -\frac{1}{3} \text{ or } x = -1}$$

5. (2 points) EXTRA CREDIT. Can a one-to-one function and its inverse be equal? If not, explain. If yes, give an example of such a function $f(x)$

Yes. For example $f(x) = x$ or $g(x) = \frac{1}{x}$. Any ~~graph~~ function whose graph is symmetric about the line $y = x$ will do.