MATH 140, Section K2
September 23, 2009

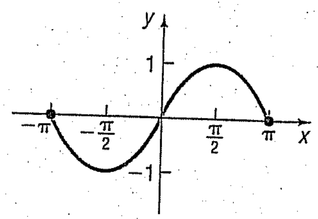
Test #1

Name: Key

READ AND FOLLOW ALL DIRECTIONS. CIRCLE YOUR FINAL ANSWERS.
SHOW ALL WORK TO RECEIVE FULL CREDIT. NO CALCULATORS.

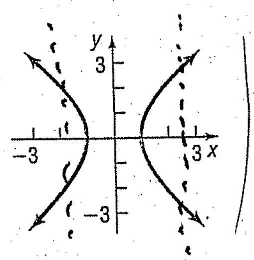
1. (8 points) Consider the following graphs. Decide if each is or is not the graph of y as a function of x. Explain your reason.

(a)



Yes, this is the graph of a function. Any vertical line intersects the graph at most once.

(b)



No, this is not the graph of a function. Several vertical lines intersect the graph twice, so it does not pass the vertical line test. (two such lines are indicated.

2. (12 points) Write a general formula for each variation.

(a) v varies directly with t; $v = 16$ when $t = 2$

$v = kt$

$16 = k \cdot 2$

$8 = k$

$\boxed{v = 8t}$

(b) F varies inversely with d^2; $F = 2$ when $d = 5$

$F = \frac{k}{d^2}$

$2 = \frac{k}{5^2}$

$50 = 5^2 \cdot 2 = k$

$\boxed{F = \frac{50}{d^2}}$

3. (14 points) Let $f(x) = 3x^2 - 4x$. Evaluate the following, and be sure to simplify your answers.

   (a) $f(-2)$

   $f(-2) = 3(-2)^2 - 4(-2)$
   $= 3 \cdot 4 + 8 = 12 + 8 = 20$

   (b) $f(x+2)$

   $f(x+2) = 3(x+2)^2 - 4(x+2)$
   $= 3(x^2+4x+4) - 4(x+2) = 3x^2+12x+12-4x-8$
   $= \boxed{3x^2+8x+4}$

   (c) $f(-x)$

   $f(-x) = 3(-x)^2 - 4(-x) = \boxed{3x^2+4x}$

4. (10 points) Let $f(x) = \frac{2x^2-3}{7-x}$ and $g(x) = \sqrt{x+5}$.

   (a) Identify the domain of f.

   $7 - x \neq 0$ so $x \neq 7$

   $\text{dom} f = \{x \mid x \neq 7\}$ or all real numbers except 7.

   (b) Identify the domain of g.

   $x + 5 \geq 0$ so $x \geq -5$

   $\text{dom}\, g = \{x \mid x \geq -5\}$ or all real numbers greater than or equal to -5

5. (10 points) For the following relations, explain why each is or is not a function.

   (a) $\{(x, y) | x = y^2\}$

   Not a function. $(1,-1)$ and $(1,1)$ are both in the relation, and in a function each element of the domain must be related to a unique element of the range.

   (b) $\{(2, -6), (-3, 6), (4, 9), (3, 4)\}$

   Function - Each element of the domain (1st coordinate) is related to a unique element of the range (2nd coordinate).

(c) The relation where each student is paired with their GPA.

This relation is a function - each student has a unique GPA.
Also acceptable: Not a function, as each GPA may be associated with more than 1 student.

6. (12 points) Consider the piecewise-defined function

$$g(x) = \begin{cases} x+3 & \text{if } -10 \le x < -6 \\ -3 & \text{if } -6 \le x < 5 \\ -2x-3 & \text{if } 5 \le x \le 10 \end{cases}$$

Evaluate:

(a) $g(-6)$

$g(-6) = -3$

(b) $g(5)$

$g(5) = -2(5) - 3 = -10 - 3 = -13$

(c) $g(-10)$

$g(-10) = -10 + 3 = -7$

7. (20 points) Let $f(x) = \sqrt{x+6}$

(a) Identify the domain of f.

$x + 6 \geq 0$ so $x \geq -6$

dom $f = \{x \mid x \geq -6\}$ or all real numbers greater than or equal to -6

(b) Is the point $(3, 3)$ on the graph of f?

$f(3) = \sqrt{3+6} = \sqrt{9} = 3$ so the point $(3,3)$ is on the graph of f.

(c) What is $f(10)$?

$f(10)=\sqrt{10+6}=\sqrt{16}=4$

(d) What are the x-intercepts, if any, of f?

$0=f(x)=\sqrt{x+6} \longleftrightarrow 0=x+6 \longleftrightarrow \boxed{x=-6}$

(e) What is the y-intercept, if any, of f?

$f(0)=\sqrt{0+6}=\sqrt{6}$

8. (14 points) Let $f(x) = x^2 + 6$ and $g(x) = \sqrt{x^3 + 3}$. Find the following functions.

(a) $(f/g)(x)$

$(f/g)(x)=\dfrac{f(x)}{g(x)}=\boxed{\dfrac{x^2+6}{\sqrt{x^3+3}}}$

(b) $(f \circ g)(x)$

$(f\circ g)(x)=f(g(x))=f(\sqrt{x^3+3})$
$=(\sqrt{x^3+3})^2+6=x^3+3+6=\boxed{x^3+9}$

9. (5 points) EXTRA CREDIT. Define the term relation. Is every relation a function? Why or why not?

A relation is a correspondence between two sets X (domain) and Y (range) or a set of ordered pairs (x, y). Some relations are not functions. For example, {(1,2), (3,4), (1,6)} is not a function because 1 is paired with two elements of the range