MATH 140, Section K2
October 21, 2009

Test #2

Name: Key

READ AND FOLLOW ALL DIRECTIONS. CIRCLE YOUR FINAL ANSWERS. SHOW ALL WORK TO RECEIVE FULL CREDIT. NO CALCULATORS.

1. (16 points) Let $f(x) = -3x^2 + 12x - 9$

   (a) Circle the correct option:

   The graph of f (opens up / opens down) and has a (highest / lowest) point.

   (b) Find the vertex of the graph of $f(x)$.

   $h = \frac{-b}{2a} = \frac{-12}{2(-3)} = \frac{-12}{-6} = 2$

   $k = f(h) = f(2) = -3\cdot 2^2 + 12\cdot 2 - 9 = -12 + 24 - 9 = 3$

   $\boxed{(h,k) = (2,3)}$

   (c) Find the x-intercept(s) of $f(x)$.

   $0 = f(x) = -3(x-2)^2 + 3$

   $-3 = -3(x-2)^2$

   $1 = (x-2)^2$

   $\pm 1 = x - 2$

   $x = 2 \pm 1$

   $\boxed{x = 1 \text{ or } 3}$

   (d) Find the y-intercept of $f(x)$

   $f(0) = -3(0)^2 + 12\cdot 0 - 9 = -9$

   $\boxed{y = -9}$

Test #2

2. (10 points) Identify the domain of each of the following functions.

   (a) $f(x) = x^{15} + 25x^5 + x^3$

   All real numbers

   (b) $R(x) = \frac{x^2-4}{x^2-x-2}$ $= \dfrac{x^2-4}{(x-2)(x+1)}$

   dom $R = \{x \mid x \neq 2, x \neq -1\}$

   (c) $s(x) = \frac{1}{\sqrt{x-4}}$

   $x - 4 > 0 \Rightarrow x > 4$

   dom $s = \{x \mid x > 4\}$

3. (16 points) The price p (in dollars) and the quantity x sold of a certain product obey the demand equation $p = \frac{-1}{10}x + 150$.

   (a) Express the revenue R as a function of x.

   $R(x) = px = \frac{-1}{10}x^2 + 150x$

   (b) What is the revenue (in dollars) if 10 units are sold?

   $R(10) = \frac{-1}{10} 10^2 + 150 \cdot 10 = -10 + 1500 = 1490$

   $\boxed{\$1490}$

   (c) What quantity x maximizes the revenue?

   $h = \frac{-b}{2a} = \dfrac{-150}{2 \cdot \frac{-1}{10}} = \dfrac{-150}{-2/10} = \dfrac{150}{1/5} = \boxed{750}$

4. (8 points) Determine whether each of the following functions is a polynomial. For those which are polynomials, state the degree.

   (a) $f(x) = x^5 + 6x + 6$

   Polynomial - degree 5

   (b) $g(x) = x^{\frac{3}{2}} + 4x + 2$

   Not a polynomial - cannot have fractional exponents

   (c) $h(x) = 15$

   Polynomial - degree 0

5. (20 points) Let $f(x) = x^3 - 3x^2 - 6x + 8$

   (a) Determine how many positive and negative real zeros $f(x)$ may have.

   $f(-x) = -x^3 - 3x^2 + 6x + 8$

   f has 2 or 0 positive real zeros, and 1 negative real zero.

   (b) List the possible rational roots of $f(x)$.

   $p: \pm 1, \pm 2, \pm 4, \pm 8$

   $q: \pm 1$

   $\boxed{\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8}$

   (c) Factor $f(x)$ completely over the real numbers.

   $f(1) = 1 - 3 - 6 + 8 = 0$

   $\boxed{f(x) = (x-1)(x-4)(x+2)}$

   $$\begin{array}{r|rrrr} 1 & 1 & -3 & -6 & 8 \\ & & 1 & -2 & -8 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$

   $f(x) = (x-1)(x^2 - 2x - 8)$

   either factor or use the quadratic formula on $x^2 - 2x - 8$ to find $x^2 - 2x - 8 = (x-4)(x+2)$

6. (20 points) Let $f(x) = (x-2)(x^2+4x+4)$

   (a) List the real zeros of $f(x)$ and their multiplicities.

   $x^2+4x+4 = (x+2)^2$

   $f(x) = (x-2)(x+2)^2$

   2 of multiplicity 1

   -2 of multiplicity 2

   (b) Identify the y-intercept of the graph of f.

   $f(0) = (0-2)(0^2+4\cdot 0+4) = (-2)(4) = -8$

   $\boxed{y=-8}$

   (c) Determine whether the graph crosses or touches the x-axis at each x-intercept. (e.g. "The graph of $f(x)$ (crosses/touches) the x-axis at x=c.")

   The graph of f crosses the x-axis at $x=2$ and touches the x-axis at $x=-2$

   (d) What power function $g(x)$ does the graph of f resemble for large values of $|x|$?

   $g(x) = x^3$

7. (10 points) Let $G(x) = \frac{x-4}{x^2-4} = \frac{x-4}{(x+2)(x-2)}$

   (a) Circle the correct option:

   $G(x)$ (is / is not) in lowest terms.

   $G(x)$ is a (proper / improper) rational function.

(b) List the vertical asymptote(s) of $G(x)$.

Vertical asymptotes are at the zeros of the denominator of $G(x)$

i.e. $\boxed{x=2 \text{ and } x=-2}$

(c) List the horizontal asymptote(s) of $G(x)$.

G is proper, so $\boxed{y=0}$ is a horizontal asymptote of $G(x)$

8. (5 points) EXTRA CREDIT. Write a quadratic function $f(x)$ which has vertex $(2,4)$ and y-intercept $(0,2)$.

$$f(x) = a(x-h)^2 + k$$
$$= a(x-2)^2 + 4$$

$$2 = f(0) = a(0-2)^2 + 4$$
$$= a(-2)^2 + 4$$
$$= 4a + 4$$

$$2 = 4a + 4$$
$$-2 = 4a$$
$$a = \frac{-2}{4} = \frac{-1}{2}$$

$$\boxed{f(x) = -\frac{1}{2}(x-2)^2 + 4}$$