MATH 140, Section K2
November 18, 2009

Test #3

Name: ____________________

READ AND FOLLOW ALL DIRECTIONS. CIRCLE YOUR FINAL ANSWERS.
SHOW ALL WORK TO RECEIVE FULL CREDIT. NO CALCULATORS.

1. (20 points) Let $f(x) = 4 - x$ and $g(x) = 1 + x^2$
   (a) Find $(f \circ g)(x)$.

   (b) Find $(g \circ f)(x)$.

   (c) Find $(f \circ f)(4)$.

   (d) Find $(g \circ g)(-1)$.

2. (7 points) Determine whether or not each of the given functions is one-to-one. Explain your reasoning.
   (a) $\{(1, 2); (3, 5); (5, 8); (6, 10)\}$

(b) $f(x) = 3x^2 + 2x + 5$

3. (15 points) The function $f(x) = \frac{2-x}{3+x}$ is one-to-one.

(a) Find its inverse function $f^{-1}(x)$.

(b) Check your answer by verifying that $(f^{-1} \circ f)(x) = x$

4. (20 points) Solve for x:

   (a) $25^{2x} = 5^{x^2-14} \cdot 25$

   (b) $\log_6 (x+3) + \log_6 (x+4) = 1$

5. (8 points) Find the exact value of each of the following expressions.

   (a) $\log_2 \left(\frac{1}{8}\right)$

   (b) $2^{\log_2 0.4}$

(c) $\ln\left(e^{\sqrt{2}}\right)$

(d) $\log_6 9 + \log_6 4$

6. (20 points) Write each expression as a single logarithm.

   (a) $-2\log_3 \frac{1}{x} + \frac{1}{3}\log_3 x^3$

   (b) $\log\left(x^2 - 1\right) - 2\log\left(x + 1\right)$

7. (10 points) For an exponential function $f(x) = a^x$, we require $a > 0$ and $a \neq 1$. Explain why this is so.

8. (5 points) EXTRA CREDIT. Prove that for $M, N > 0$, $a > 0$, $a \neq 1$, $\log_a (M \cdot N) = \log_a M + log_a N$. You may use any of the other rules for logs, except the one you are trying to prove. (Hint: Your proof should start with $\log_a (M \cdot N) = \ldots$ and follow a string of equalities to arrive at $\ldots = \log_a M + log_a N$.)