MATH 140, Section A
June 26, 2012

Test #1

Name: Key

READ AND FOLLOW ALL DIRECTIONS. CIRCLE YOUR FINAL ANSWERS. SHOW ALL WORK TO RECEIVE FULL CREDIT. NO CALCULATORS.

1. (12 points) Consider the piecewise-defined function

$$f(x) = \begin{cases} 2 - x & \text{if } 0 \leq x < 1 \\ \frac{1}{x} & \text{if } 1 \leq x < 4 \\ \sqrt{x} & \text{if } 4 \leq x \leq 9 \end{cases}$$

Evaluate:

(a) $f(0)$. Use the first rule.
$f(0) = 2 - 0 = 2$

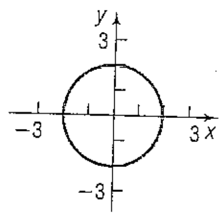
(b) $f(3)$. Use the second rule.
$f(3) = \frac{1}{3}$

(c) $f(\frac{1}{2})$. Use the first rule.
$f(\frac{1}{2}) = 2 - \frac{1}{2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$

2. (8 points) Determine whether or not each relation represents a function. Explain your reasoning.

(a) y=2x+1
The relation represents a function.
For any input x, multiply it by 2 and add 1 -
this results in a unique output.

(b)



This is not a function.
The graph fails the
vertical line test.

3. (22 points) For $f(x) = 2x^2 - 3x$, find the following

(a) $f(0)$

$f(0) = 2(0)^2 - 3\cdot 0 = 2\cdot 0 - 3\cdot 0 = 0 - 0 = 0$

(b) $f(5)$

$f(5) = 2\cdot 5^2 - 3\cdot 5 = 2\cdot 25 - 3\cdot 5 = 50 - 15 = 35$

(c) $f(x+h)$

$f(x+h) = 2(x+h)^2 - 3(x+h) = 2(x^2 + 2hx + h^2) - 3x - 3h$
$= 2x^2 + 4hx + 2h^2 - 3x - 3h = 2x^2 + (4h-3)x + 2h^2 - 3h$

(d) $f(-x)$

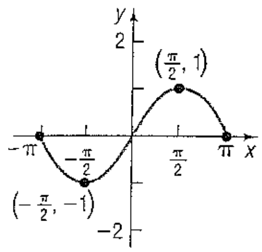
$f(-x) = 2(-x)^2 - 3(-x) = 2x^2 + 3x$

(e) Is the function f even, odd, or neither?

Neither. $f(-x) \neq f(x)$ so f is not even.
$f(-x) \neq -f(x)$, so f is not odd.

4. (8 points) Determine whether the function given is odd, even, or neither. You must provide an accurate reason to receive credit.

(a)



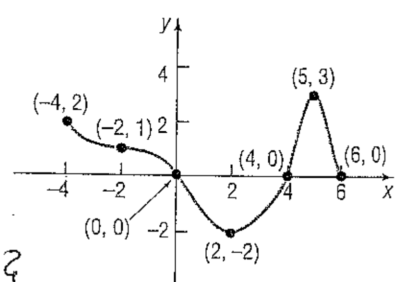
The function is odd.
Its graph is symmetric about the origin
(when rotated 180°, the same graph results)

(b) $g(x) = \sqrt{x^2+1}$

$g(-x) = \sqrt{(-x)^2+1} = \sqrt{x^2+1} = g(x)$.

The function g is even.

5. (22 points) Use the given graph of $g(x)$ to answer the following.



(a) Find $g(2)$ and $g(-2)$.

$g(2) = -2$

$g(-2) = 1$

(b) What is the domain of g?

The domain is $\{x \mid -4 \le x \le 6\}$
or $[-4, 6]$

(c) What is the range of g?

The range is $\{y \mid -2 \le y \le 3\}$
or $[-2, 3]$

(d) What are the x-intercepts of g?

$x = 0, 4,$ and 6 are the x-intercepts of g.
(The points $(0,0)$, $(4,0)$, and $(6,0)$ would also be acceptable)

(e) What is the y-intercept of g?

$y = 0$ is the y-intercept of g. (The point $(0,0)$ is also OK).

6. (12 points) Write the equation of the function whose graph is the graph of $y = x^3$, but is:

   (a) Shifted to the right 4 units.

   $y = (x-4)^3$ (Think about the x-intercept moving)

   (b) Shifted up 4 units.

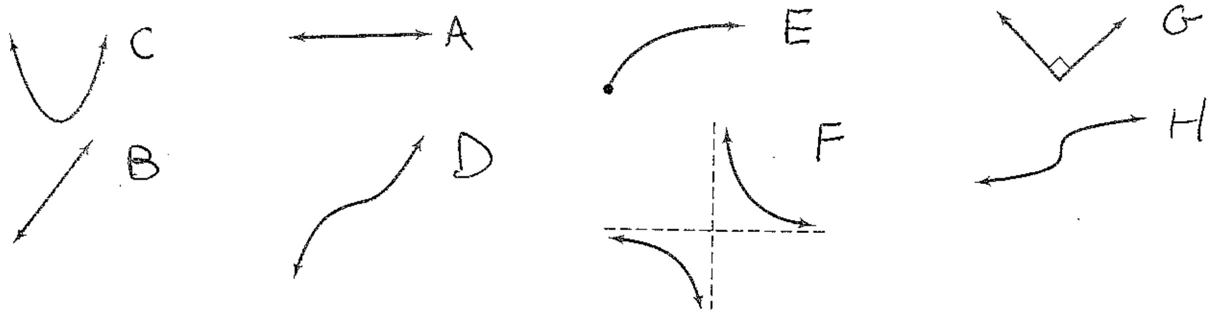
   $y = x^3 + 4$ (Think about the y-intercept moving)

   (c) Reflected over the x-axis, and vertically stretched by a factor of 4.

   $y = -4x^3$ (the 4 gives the stretch, the "−" reflects the graph over the x axis)

7. (16 points) Match each graph to the corresponding function (write the appropriate letter next to each graph).

A. Constant function *B. Identity function* *C. Square function* *D. Cube function*
E. Square root function *F. Reciprocal function* *G. Absolute value function* *H. Cube root function*



8. (5 points) EXTRA CREDIT. Name a function $f(x)$ which is both even and odd. To receive credit, prove that your function IS both even and odd.

$f(x) = 0$ is both even and odd.
$f(-x) = 0 = f(x)$, which shows that f is even.
$f(-x) = 0 = -f(x)$, which shows that f is odd.
This is the only such function.