MATH 140, Section A
July 11, 2012

Test #2

Name: Key

READ AND FOLLOW ALL DIRECTIONS. CIRCLE YOUR FINAL ANSWERS.
SHOW ALL WORK TO RECEIVE FULL CREDIT. NO CALCULATORS.

1. (8 points) Determine whether each of the following functions is a polynomial. For those which are polynomials, state the degree.

   (a) $g(x) = x^5 + 3x^2 + 2$

   Polynomial of degree 5.

   (b) $h(x) = 4(x^2+1)(x-2)^3$

   Polynomial of degree 5.

   (c) $f(x) = \frac{1}{x}$

   $\frac{1}{x} = x^{-1}$. Exponents in polynomials must be nonnegative integers, so this is not a polynomial.

2. (16 points) Let $f(x) = x^3 + 2x^2 - 5x - 6$

   (a) List the possible rational roots of $f(x)$.

   p/q: $\pm 1, \pm 2, \pm 3, \pm 6$ ($p: \pm 1, \pm 2, \pm 3, \pm 6$ (factors of a_0); $q: \pm 1$ (factors of a_3))

   (b) Factor $f(x)$ completely over the real numbers.

   $f(1) = 1+2-5-6 = -8$, $f(-1) = -1+2+5-6 = 0$ so $x+1$ a factor.

$$\begin{array}{r|l} & x^2 + x - 6 \\ \hline x+1 & x^3+2x^2-5x-6 \\ & \underline{x^3+x^2} \\ & \quad x^2-5x-6 \\ & \quad \underline{x^2+x} \\ & \quad\quad -6x-6 \\ & \quad\quad \underline{-6x-6} \\ & \quad\quad\quad 0 \end{array}$$

   So $f(x) = (x^2+x-6)(x+1)$ and x^2+x-6 factors as $(x+3)(x-2)$, thus

   $f(x) = (x+1)(x+3)(x-2)$

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3. (14 points) Let $G(x) = \frac{x^3}{x^4-1}$

   (a) Circle the correct option:
   $G(x)$ is a (proper / improper) rational function. $(\deg(x^3) < \deg(x^4-1))$

   (b) Identify the domain of $G(x)$.

   $\{x \mid x^4 - 1 \neq 0\}$ or $\{x \mid x \neq 1, x \neq -1\}$

   (c) List the vertical asymptote(s) of $G(x)$.

   The vertical asymptotes are the lines

   $\boxed{x = 1 \text{ and } x = -1.}$

   (for $c = \pm 1$, as $x \to c$, $|G(x)| \to \infty$)

   (d) List the horizontal asymptote(s) of $G(x)$.

   As $|x| \to \infty$, $G(x) \approx \frac{x^3}{x^4} = \frac{1}{x} \to 0$, thus the line $\boxed{y = 0}$ is a horizontal asymptote

4. (16 points)

   (a) Write a quadratic function $f(x)$ which has vertex $(2, 1)$ and y-intercept $(0, 5)$.

   $f(x) = a(x-h)^2 + k$ (vertex form of a quadratic)

   $f(x) = a(x-2)^2 + 1$ (substitute $(h,k) = (2,1)$)

   $5 = f(0) = a(0-2)^2 + 1 = a \cdot 4 + 1$ (since $(0,5)$ is on the graph)

   $5 = 4a + 1$

   $4 = 4a$ so $a = 1$

   $\boxed{f(x) = (x-2)^2 + 1}$ or $\boxed{f(x) = x^2 - 4x + 5}$

   (b) Fill in the blanks and circle the correct responses: The graph of $f(x)$ is that of x^2, shifted __1__ units (up / down) and __2__ units (left / right).

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5. (10 points) Identify the domain of each of the following functions.

   (a) $s(x) = \frac{1}{\sqrt{x+16}}$ $\sqrt{x+16} \neq 0$ and also $x+16 \geq 0$

   $\{x \mid x > -16\}$ or $x > -16$

   (b) $R(x) = \frac{-2x^2}{x^2-4}$

   $\{x \mid x^2 - 4 \neq 0\}$ or $\{x \mid x \neq 2, x \neq -2\}$

   (c) $f(x) = -2x^2(x^2 - 4)$

   All real numbers.

6. (20 points) Let $f(x) = (x-5)^3(x+4)^2$

   (a) List the real zeros of $f(x)$ and their multiplicities.

   5 of multiplicity 3

   -4 of multiplicity 2

   (b) Identify the y-intercept of the graph of f.

   $f(0) = (0-5)^3 \cdot (0+4)^2 = \boxed{-5^3 \cdot 4^2}$ (this is ok)

   or, simplifying $-5^3 \cdot 4^2 = -125 \cdot 16 = -2000$.

   (c) Determine whether the graph crosses or touches the x-axis at each x-intercept. (e.g. "The graph of $f(x)$ (crosses/touches) the x-axis at x=c.")

   The graph of $f(x)$ crosses the x axis at $x = 5$ (zero of odd multiplicity).

   The graph of $f(x)$ touches the x-axis at $x = -4$ (zero of even multiplicity)

   (d) What power function $g(x)$ does the graph of f resemble for large values of $|x|$?

   $g(x) = x^5$ (since this is the leading term of f).

7. (16 points) Let $f(x) = -2x^2 - 4x - 1$

(a) Circle the correct options: The graph of f (opens up / opens down) and has an absolute (maximum / minimum) point.

(b) Find the vertex of the graph of $f(x)$.

$$h = \frac{-(-4)}{2(-2)} = \frac{4}{-4} = -1 \quad (x\text{-coordinate})$$

$$k = f(h) = f(-1) = -2(-1)^2 - 4(-1) - 1$$
$$= -2 + 4 - 1 = 1$$

The vertex of f(x) is the point $(-1, 1)$

(thus the vertex form of f(x) is
$f(x) = -2(x+1)^2 + 1$)

(c) Find the y-intercept of $f(x)$.

$$f(0) = -2(0)^2 - 4(0) - 1 = \boxed{-1}$$

(d) Find the x-intercept(s), if any, of $f(x)$.

These occur where $f(x) = 0$, use the vertex form, or the quadratic formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4\cdot(-2)(-1)}}{2\cdot(-2)} = \frac{4 \pm \sqrt{16-8}}{-4} = -1 \pm \frac{\sqrt{8}}{4}$$
$$= -1 \pm \frac{2\sqrt{2}}{4} = \boxed{-1 \pm \frac{\sqrt{2}}{2}}$$

8. (5 points) EXTRA CREDIT: Write a polynomial which could have the graph below.

$$f(x) = x(x-1)^2(x-2)$$