Review: p78 #47, p145 #27, 28, 33, 39, 51, 65, 77, 79, 80
p186 #31, 41, 52

p78 #47

$$\frac{\sqrt{3}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{3}}{5-\sqrt{2}} \cdot \frac{5+\sqrt{2}}{5+\sqrt{2}} = \frac{(5+\sqrt{2})\sqrt{3}}{5^2-(\sqrt{2})^2} = \boxed{\frac{(5+\sqrt{2})\sqrt{3}}{23}}$$

p145

#27 $\sqrt{x+1} + \sqrt{x-1} = \sqrt{2x+1}$

$$(\qquad\qquad)^2 = (\sqrt{2x+1})^2$$

$$(\sqrt{x+1})^2 + 2\sqrt{x+1}\sqrt{x-1} + (\sqrt{x-1})^2 = 2x+1$$

$$x\cancel{+1} + 2\sqrt{(x+1)(x-1)} + x\cancel{-1} = 2x+1$$

$$-2x \qquad\qquad -2x$$

$$2\sqrt{(x+1)(x-1)} = 1$$

$$(x+1)(x-1) = \frac{1}{4}$$

$$x^2 - 1 = \frac{1}{4}$$

$$x^2 = \frac{5}{4} \qquad x = \pm\frac{\sqrt{5}}{2}$$

Now, check your answers.
Clearly $\sqrt{5}/2$ is OK, but if we substitute $\frac{-\sqrt{5}}{2}$, part of the left hand side is $\sqrt{\frac{-\sqrt{5}}{2}-1}$, and we cannot take the square root of a negative number. So, $\boxed{x = \frac{\sqrt{5}}{2}}$

28 $\sqrt{2x-1} - \sqrt{x-5} = 3$

$+\sqrt{x-5} \quad +\sqrt{x-5}$

$\sqrt{2x-1} = 3+\sqrt{x-5}$

$(\sqrt{2x-1})^2 = (3+\sqrt{x-5})^2$

$2x-1 = 3^2+2\cdot3\sqrt{x-5}+(\sqrt{x-5})^2$

$2x-1 = 9+6\sqrt{x-5}+x-5 -$

$2x-1 = 6\sqrt{x-5}+x+4$

$-x-4 \quad -x-4$

$x-5 = 6\sqrt{x-5}$

$(x-5)^2 = (6\sqrt{x-5})^2$

$x^2+2\cdot(-5)(x)+(-5)^2 = 36(x-5)$

$x^2-10x+25 = 36x-180$

$-36x+180 \quad -36x+180$

$x^2-46x+205 = 0$

$$x = \frac{-(-46)\pm\sqrt{(-46)^2-4\cdot1\cdot205}}{2\cdot1} = \frac{46\pm\sqrt{2116-820}}{2}$$

$$x = \frac{46\pm\sqrt{1296}}{2} = \frac{46\pm36}{2} = \frac{82}{2} \text{ or } \frac{10}{2} = \boxed{41 \text{ or } 5}$$

Neither solution makes the term underneath the square root negative, so both are acceptable.

P145 #33

$$x^2+m^2 = 2mx + (nx)^2 \qquad n \neq 1$$

$$x^2+m^2 = 2mx + n^2x^2$$

$$0 = (1-n^2)x^2 + (2m)x + m^2$$

$$x = \frac{-2m \pm \sqrt{(2m)^2-4(1-n^2)m^2}}{2(1-n^2)} = \frac{-2m \pm \sqrt{4m^2-4m^2(1-n^2)}}{2(1-n^2)}$$

$$= \frac{-\cancel{2}m \pm \cancel{2}m\sqrt{1-(1-n^2)}}{\cancel{2}(1-n^2)} = \frac{-m \pm m\sqrt{1-1+n^2}}{1-n^2}$$

$$x = \frac{-m \pm 2m\sqrt{n^2}}{1-n^2} = \frac{-m \pm 2mn}{1-n^2}$$

#39 $|2x+3| = 7$

$$2x+3 = 7 \quad \text{or} \quad 2x+3 = -7$$

$$2x = 4 \quad \text{or} \quad 2x = -10$$

$$\boxed{x = 2 \quad \text{or} \quad x = -5}$$

#51 $2 < \frac{3-3x}{12} < 6$

$$12 \cdot 2 < 3-3x < 12 \cdot 6$$

$$24 < 3-3x < 72$$

$$24-3 < -3x < 72-3$$

$$21 < -3x < 69$$

$$\frac{-21}{3} > x > \frac{-69}{3}$$

$$-7 > x > -23$$ (remember to flip the inequality)

$$\boxed{-23 < x < -7}$$

p 145

#65 $\frac{3}{3+i}$ The procedure is much like rationalizing the denominator.

$$\frac{3}{3+i}\cdot\frac{3-i}{3-i}=\frac{3(3-i)}{3^2-i^2}=\frac{3(3-i)}{9-(-1)}=\frac{9-3i}{9+1}=\frac{9-3i}{10}$$

$$=\frac{9}{10}-\frac{3}{10}i$$

#77 Solve:

$$x(1-x)=6$$

$$x(1)+x(-x)=6$$

$$x-x^2=6$$

$$-x+x^2 \quad -x+x^2$$

$$0=x^2-x+6$$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot1\cdot6}}{2\cdot1}=\frac{1\pm\sqrt{1-24}}{2}=\frac{1\pm\sqrt{-23}}{2}=\frac{1\pm i\sqrt{23}}{2}$$

$$\boxed{x=\frac{1}{2}+\frac{i\sqrt{23}}{2} \text{ or } x=\frac{1}{2}-\frac{i\sqrt{23}}{2}}$$

#79 The perimeter p of a rectangle is the sum of two times the length l and two times the width w

$$\boxed{p=2l+2w}$$

#80 The total cost C of manufacturing x bicycles in one day is \$50,000 plus \$95 times the number of bicycles manufactured.

$$C=50000+95x$$

p186

#31 $2x^2+2y^2-12x+8y-24=0$

(a) We solve by completing the square

$$(2x^2-12x)+(2y^2+8y)-24=0$$

$$2(x^2-6x)+2(y^2+4y)-2(12)=0$$

$$2\left(x^2-6x+\left(\tfrac{-6}{2}\right)^2\right)+2\left(y^2+4y+\left(\tfrac{4}{2}\right)^2\right)-24-2\cdot\left(\tfrac{-6}{2}\right)^2-2\left(\tfrac{4}{2}\right)^2=0$$

$$2(x^2-6x+9)+2(y^2+4y+4)-24-2\cdot9-2\cdot4=0$$

$$2(x-3)^2+2(y+2)^2-24-18-8=0$$

$$2(x-3)^2+2(y+2)^2-50=0$$
$$+50 \quad +50$$

$$\frac{2(x-3)^2+2(y+2)^2=50}{2}$$

$$(x-3)^2+(y+2)^2=25=5^2$$

$$(h,k)=(3,-2)$$

(b) The circle will be centered at (3,2) and have radius 5

(c) The y intercept(s) occur(s) when $x=0$

$$(0-3)^2+(y+2)^2=25$$

$$9+(y+2)^2=25 \longrightarrow (y+2)^2=16 \longrightarrow y+2=\pm4 \quad \boxed{y=6 \text{ or } -2}$$

The x intercept(s) occur(s) when $y=0$

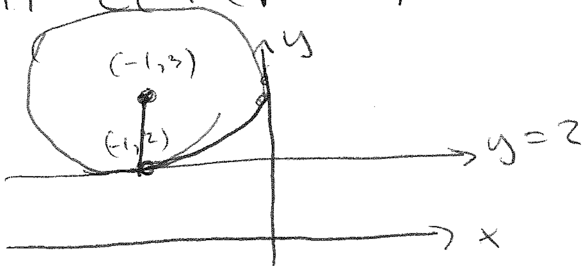
$$(x-3)^2+(0+2)^2=25$$

$$(x-3)^2+4=25$$

$$(x-3)^2=21 \longrightarrow x-3=\pm\sqrt{21} \longrightarrow \boxed{x=3\pm\sqrt{21}}$$

p 186

#41 center $(-1,3)$ and tangent to $y=2$



(sketch of graph

The radius will be $\underline{\underline{1}}$, and in standard form
a circle has equation

$(x-h)^2+(y-k)^2=r^2$ so this one is

$(x-(-1))^2+(y-3)^2=1^2$

$$\boxed{(x+1)^2+(y-3)^2=1^2}$$

#52

(a) $x^2+(mx+b)^2=r^2$ has exactly one solution, so...

$x^2+(mx)^2+2bmx+b^2=r^2$

$(1+m^2)x^2+2bmx+(b^2-r^2)=0$ has discriminant equal to 0

i.e. $(2bm)^2-4(1+m^2)(b^2-r^2)=0$

$\cancel{4}b^2m^2-\cancel{4}(b^2-r^2+m^2b^2-m^2r^2)=0$

$b^2m^2-(b^2-r^2+m^2b^2-m^2r^2)=0$

$b^2m^2-b^2+r^2-m^2b^2+m^2r^2=0$

$\cancel{b^2m^2-b^2m^2}+r^2-b^2+m^2r^2=0$

$r^2+m^2r^2-b^2=0$

$$\boxed{r^2(1+m^2)=b^2}$$

Skip (c)

p 186

#52

(b) the point of tangency is $\left(\frac{-r^2m}{b}, \frac{r^2}{b}\right)$

well if $x = \frac{-r^2m}{b}$, substitution into the line's eqn gives

$$y = m\left(\frac{-r^2m}{b}\right) + b$$

$$= \frac{-r^2m^2}{b} + b$$

$$= \frac{b^2 - r^2m^2}{b} = \frac{r^2(1+m^2) - r^2m^2}{b} \quad \text{(using (a))}$$

$$= \frac{r^2 + r^2m^2 - \cancel{r^2}m^2}{b} = \frac{r^2}{b}$$

Likewise, $x = \frac{-r^2m}{b}$ in the circle's eqn gives

$$\left(\frac{-r^2m}{b}\right)^2 + y^2 = r^2$$

$$y^2 = r^2 - \left(\frac{-r^2m}{b}\right)^2$$

$$= r^2 - \frac{r^4m^2}{b^2} = r^2\left(1 - \frac{r^2m^2}{b^2}\right)$$

$$= r^2\left(1 - \frac{\cancel{r^2}m^2}{\cancel{r^2}(1+m^2)}\right) \quad \text{using (a)}$$

$$= r^2\left(\frac{1+m^2}{1+m^2} - \frac{m^2}{1+m^2}\right) = \frac{r^2}{1+m^2} = \frac{r^2}{b^2/r^2} = \frac{r^4}{b^2}$$

$$y^2 = \frac{r^4}{b^2} \iff y = \pm\frac{r^2}{b}$$

(c) <u>Hint</u>: perpendicular lines have opposite, reciprocal slopes.