Section 2.5 3-17 odd, 21, 25, 35, 36, 40, 44

3. $y = kx \rightarrow 2 = k \cdot 10 \rightarrow k = \frac{2}{10} = \frac{1}{5} \rightarrow \boxed{y = \frac{1}{5}x}$

5. $A = kx^2 \longrightarrow 4\pi = k \cdot 2^2 = 4k \longrightarrow k = \pi \rightarrow \boxed{A = \pi x^2}$

7. $F = \frac{k}{d^2} \longrightarrow 10 = \frac{k}{5^2} \longrightarrow k = 25 \cdot 10 = 250 \rightarrow \boxed{F = \frac{250}{d^2}}$

9. $z = k(x^2+y^2) \rightarrow 5 = k(3^2+4^2) = k \cdot 25 \longrightarrow k = \frac{5}{25} = \frac{1}{5}$
$\boxed{z = \frac{1}{5}(x^2+y^2)}$

11. $M = \frac{kd^2}{\sqrt{x}} \longrightarrow 24 = \frac{k \cdot 4^2}{\sqrt{9}} = \frac{16k}{3} \rightarrow k = \frac{3 \cdot 24}{16} = \frac{9}{2}$
$\boxed{M = \frac{9d^2}{2\sqrt{x}}}$

13. $T^2 = \frac{ka^3}{d^2} \longrightarrow 2^2 = \frac{k \cdot 2^3}{4^2} \longrightarrow k = 8 \rightarrow \boxed{T^2 = \frac{8a^3}{d^2}}$

15. $V = kr^3$, $k = \frac{4\pi}{3}$ so $\boxed{V = \frac{4\pi}{3}r^3}$

17. $A = kbh$, $k = \frac{1}{2}$ so $\boxed{A = \frac{1}{2}bh}$

19. $F = \frac{GmM}{d^2}$, $G = 6.67 \cdot 10^{-11}$ so $\boxed{F = 6.67 \cdot 10^{-11} \frac{mM}{d^2}}$

21. $p = kB \longrightarrow 6.49 = k \cdot 1000$ so $k = .00649$
$p = 0.00649B$
If $B = \$145,000$ then
$p = 0.00649 \cdot 145,000 = \boxed{941.05}$

25. $E = kW$ $\quad\quad E = \frac{3}{20}W$

$3 = k\cdot 20$ so and when $W=15$

$k = 3/20$ $\quad\quad E = \frac{3\cdot 15}{20} = \frac{45}{20} = 2.25$

35. $V = kr^2h$

k = constant of proportionality = π

$\boxed{V = \pi r^2 h}$

36. $V = kr^2h$

k = constant of proportionality = $\pi/3$

$\boxed{V = \frac{\pi}{3} r^2 h}$

40. $V = \frac{kT}{P}$

$100 = \frac{k\cdot 300}{15} \longrightarrow k = \frac{100\cdot 15}{300} = 5$

$V = \frac{5T}{P} \rightarrow 80 = \frac{5\cdot 310}{P} \longrightarrow P = \frac{5\cdot 310}{80} = \boxed{19.375 \text{ atm}}$

44. $M = \frac{kwt^2}{L}$

$750 = \frac{k\cdot 4\cdot 2^2}{8} \longrightarrow k = \frac{750\cdot 8}{4\cdot 2^2} = 375$

$M = \frac{375wt^2}{L} \longrightarrow M = \frac{375\cdot 6\cdot 2^2}{10} = \boxed{900 \text{ pounds}}$

Section 3.1 17–69 EOO, 43, 48, 59, 64, 75, 81

17. Not a function. The input "20 hours" corresponds to the outputs \$200 and \$300

21. This is a function. Every input corresponds to a unique output. Domain: ~~$\{-2, -1, 0, 1\}$~~ $\{1, 2, 3, 4\}$
Range: $\{3\}$

25. This is a function.
Each input corresponds to a unique output.
Domain: $\{-2, -1, 0, 1\}$
Range: $\{0, 1, 4\}$

29. $y = \frac{1}{x}$ defines y as a function of x.

33. $x = y^2$ does not define y as a function of x. If we solve for y explicitly, we have $y = \pm\sqrt{x}$, so each input (value of x) corresponds to two outputs.

37. $2x^2 + 3y^2 = 1$ does not define y as a function of x. If we solve for y explicitly, we have $y = \pm\sqrt{\frac{1-2x^2}{3}}$, so each input corresponds to two outputs.

41. $f(x) = \frac{x}{x^2+1}$

(a) $f(0) = \frac{0}{0^2+1} = 0$

(b) $f(1) = \frac{1}{1^2+1} = \frac{1}{2}$

(c) $f(-1) = \frac{-1}{(-1)^2+1} = \frac{-1}{2}$

(d) $f(-x) = \frac{-x}{(-x)^2+1} = \frac{-x}{x^2+1}$

(e) $-f(x) = \frac{-x}{x^2+1}$

(f) $f(x+1) = \frac{x+1}{(x+1)^2+1} = \frac{x+1}{x^2+2x+2}$

(g) $f(2x) = \frac{2x}{(2x)^2+1} = \frac{2x}{4x^2+1}$

(h) $f(x+h) = \frac{x+h}{(x+h)^2+1}$
$= \frac{x+h}{x^2+2hx+h^2+1}$

45. $f(x) = \dfrac{2x+1}{3x-5}$

(a) $f(0) = \dfrac{2\cdot 0+1}{3\cdot 0-5} = \dfrac{-1}{5}$

(b) $f(1) = \dfrac{2\cdot 1+1}{3\cdot 1-5} = -\dfrac{3}{2}$

(c) $f(-1) = \dfrac{2(-1)+1}{3(-1)-5} = \dfrac{-1}{-8} = \dfrac{1}{8}$

(d) $f(-x) = \dfrac{2(-x)+1}{3(-x)-5} = \dfrac{-2x+1}{-3x-5}$

(e) $-f(x) = \dfrac{-(2x+1)}{3x-5} = \dfrac{-2x-1}{3x-5}$

(f) $f(x+1) = \dfrac{2(x+1)+1}{3(x+1)-5} = \dfrac{2x+3}{3x-2}$

(g) $f(2x) = \dfrac{2(2x)+1}{3(2x)-5} = \dfrac{4x+1}{6x-5}$

(h) $f(x+h) = \dfrac{2(x+h)+1}{3(x+h)-5} = \dfrac{2x+2h+1}{3x+3h-5}$

49. The domain of $f(x) = \dfrac{x}{x^2+1}$ is
$\{x \mid x^2+1 \neq 0\}$. But x^2+1 is never 0, so the domain is the set of all real numbers.

53. The domain of $F(x) = \dfrac{x-2}{x^3+x}$ is
$\{x \mid x^3+x \neq 0\}$ i.e. $\{x \mid x \neq 0\}$

57. The domain of $f(x) = \dfrac{4}{\sqrt{x-9}}$ is
$\{x \mid x-9 > 0\}$ i.e. $\{x \mid x > 9\}$
(careful here! $x-9 \geq 0$ since it is under an even root, but it is <u>also</u> a denominator, so $x-9 \neq 0$.)

61. The domain of $p(t) = \dfrac{\sqrt{t-4}}{3t-21}$ is
$\{x \mid t-4 \geq 0 \text{ and } 3t-21 \neq 0\}$ i.e.
$\{x \mid t \geq 4 \text{ and } t \neq 7\}$

Section 3.1 continued

43. $f(x) = |x| + 4$

(a) $f(0) = |0| + 4 = 4$

(b) $f(1) = |1| + 4 = 5$

(c) $f(-1) = |-1| + 4 = 5$

(d) $f(-x) = |-x| + 4 = |x| + 4$

(e) $-f(x) = -|x| - 4$

(f) $f(x+1) = |x+1| + 4$

(g) $f(2x) = |2x| + 4 = 2|x| + 4$

(h) $f(x+h) = |x+h| + 4$

(f) and (h) cannot be simplified, Why?

48. The domain of $f(x) = x^2 + 2$ is $\mathbb{R}$.

59. The domain of $p(x) = \sqrt{\frac{2}{x-1}}$ is

$\{x \mid \frac{2}{x-1} \geq 0 \text{ and } x - 1 \neq 0\}$

But $\frac{2}{x-1} \geq 0$ precisely when $x - 1 \geq 0$, i.e. $x \geq 1$.

So the domain is

$\{x \mid x \geq 1 \text{ and } x \neq 1\}$ i.e. $\boxed{\{x \mid x > 1\}}$

64. $f(x) = 2x + 1$; $g(x) = 3x - 2$

(a) $(f+g)(x) = 5x - 1$ with domain $\mathbb{R}$

(b) $(f-g)(x) = -x + 3$ with domain $\mathbb{R}$

(c) $(f \cdot g)(x) = (2x+1)(3x-2) = 6x^2 - x - 2$ with domain $\mathbb{R}$

(d) $(f/g)(x) = \frac{2x+1}{3x-2}$ with domain $\{x \mid 3x - 2 \neq 0\}$ i.e. $\{x \mid x \neq \frac{2}{3}\}$

(e) $(f+g)(3) = 5(3) - 1 = 14$

(f) $(f-g)(4) = -4 + 3 = -1$

(g) $(f \cdot g)(2) = 6 \cdot 2^2 - 2 - 2 = 6 \cdot 4 - 4 = 20$

(h) $\left(\frac{f}{g}\right)(1) = \frac{2 \cdot 1 + 1}{3 \cdot 1 - 2} = \frac{3}{1} = 3$

75. $f(x) = 4x + 3$

$$\frac{f(x+h) - f(x)}{h} = \frac{(4(x+h)+3) - (4x+3)}{h}$$

$$= \frac{\cancel{4x} + 4h \cancel{+3} \cancel{-4x} \cancel{-3}}{h} = \frac{4h}{h} = \boxed{4}$$

81. $f(x) = \sqrt{x}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$ (use the hint)

$$= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \boxed{\frac{1}{\sqrt{x+h} + \sqrt{x}}}$$

Section 3.2 9-27 odd, 26, 28, 39

9 (a) $f(0)=3$; $f(-6)=-3$
(b) $f(6)=0$; $f(11)=1$
(c) $f(3)$ is positive
(d) $f(-4)$ is negative
(e) $f(x)=0$ when $x=-3, 6,$ and 10
(f) $f(x)>0$ when $-3<x<6$ and $10<x\le 11$
(g) the domain is $\{x \mid -6\le x\le 11\}$
(h) the range of f is $\{y \mid -3\le y\le 4\}$
(i) x-intercepts: $-3, 6, 10$
(j) y-intercept: 3
(k)-(n) On last page

11. This graph fails the vertical line test, so it is not the graph of a function.

13. This is the graph of a function
(a) domain is $[-\pi, \pi]$; range is $[-1, 1]$
(b) x intercepts: $\frac{\pi}{2}$ and $-\frac{\pi}{2}$; y-intercept: 1
(c) symmetric w.r.t the y-axis.

15. This graph fails the vertical line test, so it is not the graph of a function.

17. This is the graph of a function.
(a) domain: $x>0$; range: $\mathbb{R}$
(b) x-intercept: 1; y-intercept: none
(c) No symmetry

19. This is the graph of a function.
(a) domain: All real numbers; range: $y\le 2$
(b) x-intercepts: $3, -3$; y-intercept: 2
(c) symmetric wrt y-axis

21. This is the graph of a function

(a) ~~domain: {x | x ≥ 2~~ domain: $\mathbb{R}$; range: $\{y \mid y \geq 2\}$

(b) x-intercepts: 1, 3; y-intercept: 9

(c) No symmetry

23. $f(x) = 2x^2 - x - 1$

(a) $f(-1) = 2(-1)^2 - (-1) - 1 = 2 + 1 - 1 = 2$, so $(-1, 2)$ is on the graph

(b) $F(x)$ when $x = -2$ is $f(-2) = 2(-2)^2 - (-2) - 1 = 2 \cdot 4 + 2 - 1 = 5$
$(-2, 5)$ is on the graph of f.

(c) $f(x) = -1 \Rightarrow 2x^2 - x - \cancel{1} = \cancel{-1} \Rightarrow 2x^2 - x = 0 \rightarrow (2x - 1)x = 0$
so $x = \frac{1}{2}$ or $x = 0$. $(\frac{1}{2}, -1)$ and $(0, -1)$ are on the graph.

(d) The domain is the set of all real numbers.

(e) x-intercepts occur when $f(x) = 0$, i.e. $2x^2 - x - 1 = 0$
so $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} = \frac{1 \pm \sqrt{9}}{4} = \frac{1 \pm 3}{4} = \boxed{1 \text{ or } \frac{-1}{2}}$

(f) The y intercept is $f(0) = 2(0)^2 - 0 - 1 = \boxed{-1}$

25. $f(x) = \frac{x+2}{x-6}$

(a) $f(3) = \frac{3+2}{3-6} = \frac{5}{-3} = -\frac{5}{3}$, so $(3, 14)$ is not on the graph

(b) $f(x)$ when $x = 4$ is $f(4) = \frac{4+2}{4-6} = \frac{6}{-2} = \boxed{-3}$
so, $(4, -3)$ is on the graph of f.

(c) $f(x) = 2$ when $2 = \frac{x+2}{x-6}$, i.e. $2(x-6) = x + 2 \rightarrow 2x - 12 = x + 2$
$\boxed{x = 14}$, so $(14, 2)$ is on the graph

(d) The domain is $\{x \mid x \neq 6\}$

(e) The x intercept is when $f(x) = 0$, i.e. $\frac{x+2}{x-6} = 0$
so, at $\boxed{x = -2}$

(f) The y-intercept is $f(0) = \frac{0+2}{0-6} = \frac{2}{-6} = \boxed{\frac{-1}{3}}$

Section 3.2 continued

27. $f(x) = \frac{2x^2}{x^4+1}$

(a) $f(-1) = \frac{2(-1)^2}{(-1)^4+1} = \frac{2}{2} = 1$, so $(-1, 1)$ is on the graph

(b) $f(2) = \frac{2 \cdot 2^2}{2^4+1} = \frac{8}{16+1} = \frac{8}{17}$ so $(2, \frac{8}{17})$ is on the graph

(c) $f(x)=1$ when $1=(2x^2)/(x^4+1)$ i.e. $x^4+1=2x^2$, so at $x=\pm 1$ $(-1,1)$ and $(1,1)$ are on the graph.

(d) The domain is the set of all real numbers.

(e) The x intercept is when $f(x)=0$, i.e. at $x=0$

(f) The y intercept is $f(0) = \frac{2 \cdot 0^2}{0^4+1} = 0$

26. $f(x) = \frac{x^2+2}{x+4}$

(a) $f(1) = \frac{1^2+2}{1+4} = \frac{3}{5}$ so $(1, \frac{3}{5})$ is on the graph

(b) $f(0) = \frac{0^2+2}{0+4} = \frac{2}{4} = \frac{1}{2}$ so $(0, \frac{1}{2})$ is on the graph

(c) $\frac{1}{2} = \frac{x^2+2}{x+4} \rightarrow \frac{1}{2}(x+4) = x^2+2 \rightarrow x+4 = 2x^2+4$
$\rightarrow 2x^2 - x = 0 \rightarrow (2\cancel{x}-1)x = 0 \rightarrow x = \frac{1}{2}$ or 0; $(0, \frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{2})$ are on the graph

(d) the domain is $\{x \mid x+4 \neq 0\}$ i.e. $\{x \mid x \neq -4\}$

(e) The graph has no x-intercepts, as $f(x)=0$ when $x^2+2=0$, but this is never the case.

(f) from (b), $f(0)=\frac{1}{2}$, so the y-intercept is $\frac{1}{2}$

28. $f(x) = \dfrac{2x}{x-2}$

(a) $f(\tfrac{1}{2}) = \dfrac{2\cdot\frac{1}{2}}{\frac{1}{2}-2} = \dfrac{1}{\frac{-3}{2}} = -\dfrac{2}{3}$ so $(\tfrac{1}{2}, -\tfrac{2}{3})$ is on the graph

(b) $f(4) = \dfrac{2\cdot 4}{4-2} = \dfrac{8}{2} = 4$; $(4,4)$ is on the graph.

(c) $f(x) = 1 = \dfrac{2x}{x-2} \rightarrow x-2 = 2x \rightarrow x = -2$

so $(-2,1)$ is on the graph

(d) The domain is $\{x \mid x-2 \neq 0\}$ i.e. $\{x \mid x \neq 2\}$

(e) The x intercept is where $f(x) = 0$, i.e. $\dfrac{2x}{x-2} = 0$.

so the x-intercept is $x = 0$

(f) The y intercept is $f(0) = \dfrac{2\cdot 0}{0-2} = 0$

39. (a) $\underline{\text{III}}$ (c) I (e) II

(b) IV (d) V

9 (k) The line $y = \frac{1}{2}$ intersects the graph 3 times

(l) The line $x = 5$ intersects the graph <u>once</u>,

otherwise it would not be the graph of a function

(m) $f(x) = 3$ at $x = 0$ and $x = 4$

(n) $f(x) = -2$ at $x = -5$ and $x = 8$.