Section 4.3 # 11-18, 19, 23, 27, 47-52, 75

11. C (y-int −1, opens up)

12. E (y-int −1, opens down)

13. F (vertex (1, 0))

14. A (vertex (−1, 0))

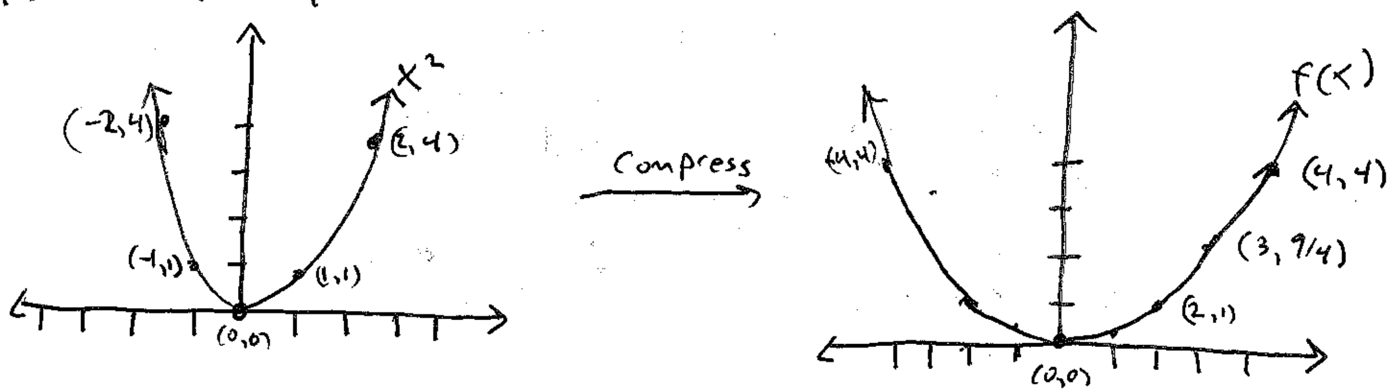
15. G (vertex (1, 1))

16. B (vertex (−1, −1) y-int (0, 0))

17. H (vertex (1, −1) y-int (0, 0))

18. D (vertex (−1, 1) y-int (0, 2))

19. $f(x) = \frac{1}{4}x^2$

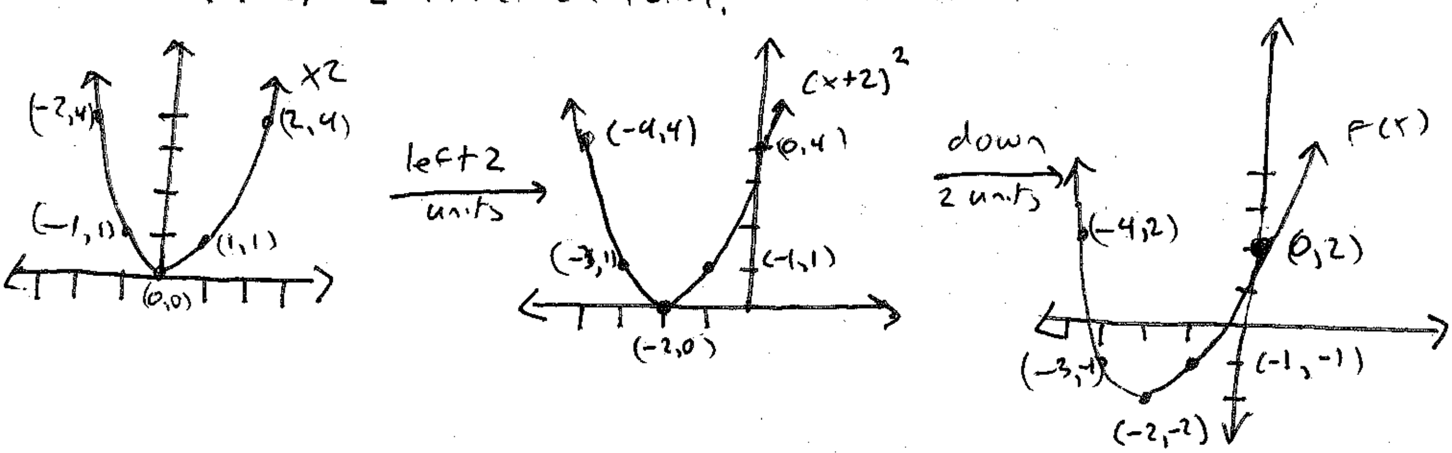


23. $f(x) = x^2 + 4x + 2$

$h = -b/2a = -4/(2\cdot 1) = -2$

$k = f(h) = f(-2) = (-2)^2 + 4(-2) + 2 = 4 - 8 + 2 = -2$

$f(x) = (x+2)^2 - 2$ in vertex form.

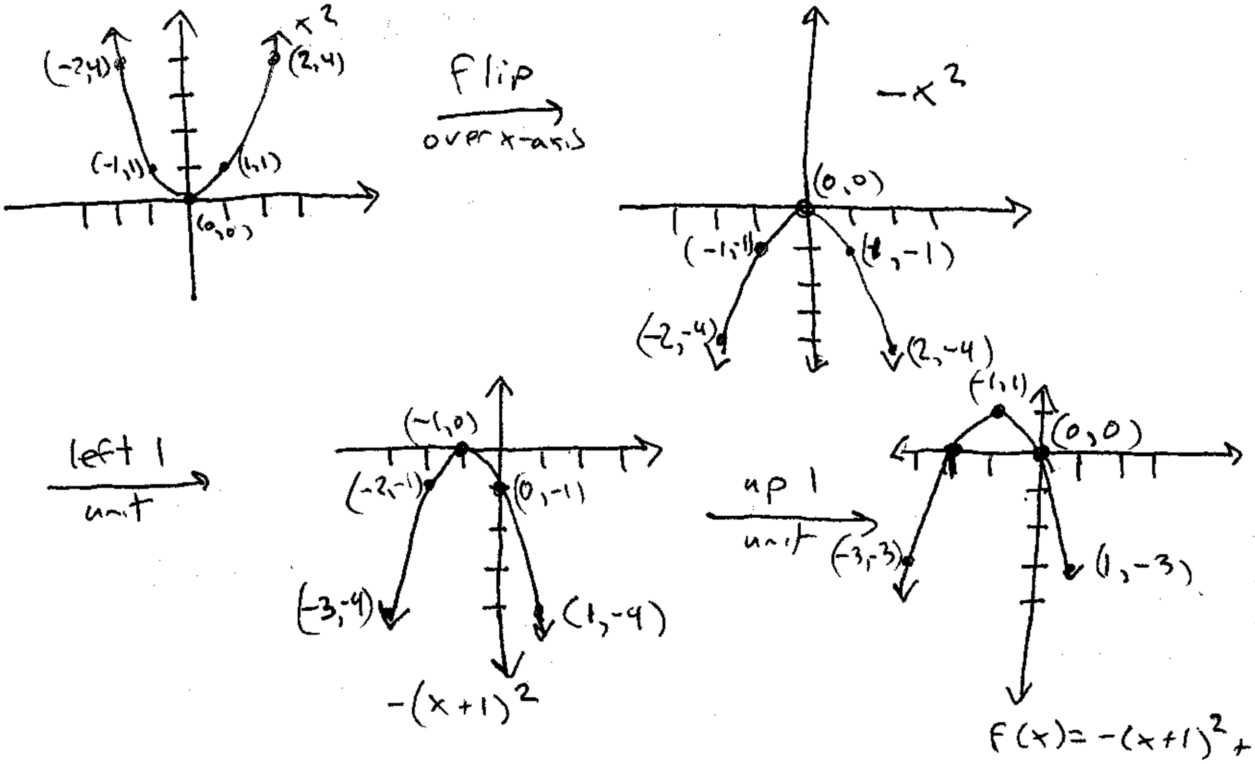


27. $f(x) = -x^2 - 2x$

$h = \frac{-b}{2a} = \frac{-(-2)}{2(-1)} = \frac{2}{-2} = -1$

$k = f(h) = f(-1) = -(-1)^2 - 2(-1) = -1 + 2 = 1$

$f(x) = -(x+1)^2 + 1$ in vertex form



47. vertex $(-1,-2)$ y-intercept $(0,-1)$

$f(x) = a(x - -1)^2 + -2 = a(x+1)^2 - 2$ using $(h,k) = (-1,-2)$

$-1 = f(0) = a(0+1)^2 - 2 = a - 2 \Rightarrow a = 1$ using the y-int.

$f(x) = (x+1)^2 - 2 = (x^2 + 2x + 1) - 2 = x^2 + 2x - 1$

48. vertex $(2,1)$ y-intercept $(0,5)$

$f(x) = a(x-2)^2 + 1$ using $(h,k) = (2,1)$

$5 = f(0) = a(0-2)^2 + 1 = a \cdot 4 + 1 \Rightarrow 4a = 4 \Rightarrow a = 1$ using y-int.

$f(x) = (x-2)^2 + 1 = (x^2 - 4x + 4) + 1 = x^2 - 4x + 5$

49. vertex $(-3,5)$ y-intercept $(0,-4)$

$f(x) = a(x - -3)^2 + 5 = a(x+3)^2 + 5$ using $(h,k) = (-3,5)$

$-4 = f(0) = a(0+3)^2 + 5 = 9a + 5 \Rightarrow 9a = -9 \Rightarrow a = -1$ using y-int.

$f(x) = -(x+3)^2 + 5 = -(x^2 + 6x + 9) + 5 = -x^2 - 6x - 4$

50. vertex $(2,3)$ y-intercept $(0,-1)$

$f(x) = a(x-2)^2+3$ using $(h,k)=(2,3)$

$-1 = f(0) = a(0-2)^2+3 = 4a+3 \Rightarrow 4a=-4 \Rightarrow a=-1$ using y-int.

$f(x) = -(x-2)^2+3 = -(x^2-4x+4)+3 = -x^2+4x-1$

51. vertex $(1,-3)$; additional point $(3,5)$

$f(x) = a(x-1)^2+-3 = a(x-1)^2-3$ using $(h,k) = 1,-3)$

$5 = f(3) = a(3-1)^2-3 = 4a-3 \Rightarrow 4a=8 \Rightarrow a=2$

using the point $(3,5)$, which tells us $f(3)=5$

$f(x) = 2(x-1)^2-3 = 2(x^2-2x+1)-3 = 2x^2-4x-1$

52. vertex $(-2,6)$; additional point $(-4,-2)$

$f(x) = a(x--2)^2+6 = a(x+2)^2+6$ using $(h,k)=(-2,6)$

$-2 = f(-4) = a(-4+2)^2+6 = 4a+6 \Rightarrow 4a=-8 \Rightarrow a=-2$

using the point $(-4,-2)$, which tells us $f(-4)=-2$.

$f(x) = -2(x+2)^2+6 = -2(x^2+4x+4)+6$

$= -2x^2-8x-2$

76. Revenue as a function of price

$R(p) = -\frac{1}{2}p^2+1900p$

The parabola/graph of R opens down and has an absolute maximum value at the vertex (h,k) where:

$h = \frac{-b}{2 \cdot a} = \frac{-1900}{2\left(\frac{-1}{2}\right)} = \frac{-1900}{-1} = 1900$ dollars

$k = f(1900) = -\frac{1}{2}(1900)^2+1900 \cdot 1900 = \frac{1}{2}(1900)^2$

$= 1,805,000$

So, they should charge \$1900 per mower to achieve a maximum revenue of \$1,805,000.

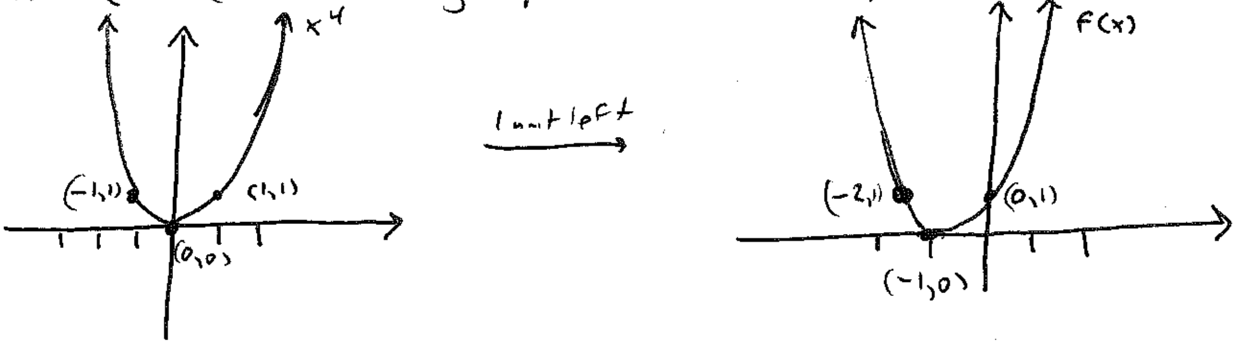
Section 5.1 #15-59 EOO, 61-69

15. $f(x) = 4x + x^3$ is a polynomial function of degree 3.

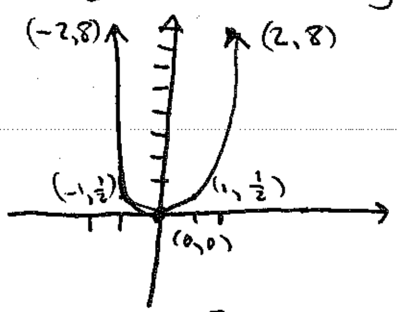
19. $f(x) = 1 - \frac{1}{x} = 1 - x^{-1}$ is not a polynomial. All of the exponents must be nonnegative integers.

23. $F(x) = 5x^4 - \pi x^3 + \frac{1}{2}$ is a polynomial function of degree 4.

27. $f(x) = (x+1)^4$ has graph similar to x^4, shifted unit left.

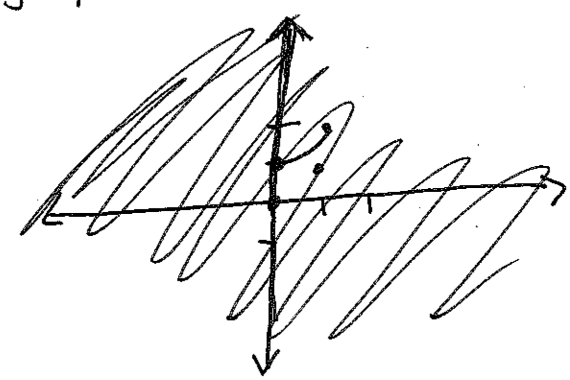


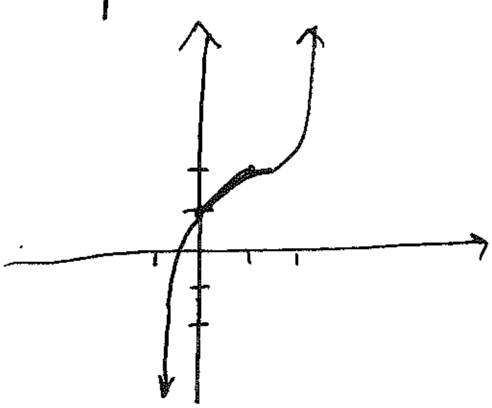
31. $f(x) = \frac{1}{2}x^4$ has the graph of x^4, compressed by a factor of $\frac{1}{2}$



35. $f(x) = (x-1)^5 + 2$ has the graph of x^5, shifted 1 unit right and 2 units up

x	x^5	$f(x)$
-1	-1	-30
0	0	1
1	1	2
2	32	3





39. $f(x) = 4-(x-2)^5$ has the graph of x^5, reflected over the x-axis, shifted right 2 units, and shifted up 4 units.

43. zeros: $-3, 0, 4$ degree: 3

$$f(x) = (x+3)(x)(x-4)$$
$$= (x+3)(x^2-4x)$$
$$= x^3+3x^2-4x^2-12x$$
$$f(x) = x^3-x^2-12x$$

47. zeros: -1 of mult. 1; 3 of mult 2; degree 3

$$f(x) = (x+1)(x-3)^2$$
$$= (x+1)(x^2-6x+9)$$
$$= x^3-6x^2+9x+x^2-6x+9$$
$$f(x) = x^3-5x^2+3x+9$$

51. $f(x) = 4(x^2+1)(x-2)^3$

(a) f has only one real zero: 2 of multiplicity 3.
(The factor x^2+1 has no real zeros; it has discriminant -4)

(b) The graph will cross the x-axis at $x=2$

(d) The polynomial f has degree 5. The graph of f has at most 4 turning points.

(e) The graph of f resembles that of $4x^5$ for large values of $|x|$

55. $f(x) = (x-5)^3(x+4)^2$

(a) f has two real zeros:
5 of multiplicity 3 <u>and</u> -4 of multiplicity 2.

(b) The graph will cross the x-axis at $x=5$ and touch the x-axis at $x=-4$.

(d) The polynomial f has degree 5. The graph of f has at most 4 turning points.

(e). The graph of f resembles that of $4x^5$ for large values of $|x|$.

59. $f(x) = -2x^2(x^2-2) = -2x^2(x+\sqrt{2})(x-\sqrt{2})$

(a) f has 3 real zeros:
0 of multiplicity 2, $\sqrt{2}$ of multiplicity 1, and $-\sqrt{2}$ of multiplicity 1.

(b) The graph of f will touch the x axis at $x=0$, and it will cross the x axis at $x=\pm\sqrt{2}$.

(d). The polynomial f has degree 4, so the graph of f will have at most 3 turning points,

(e) The graph of f resembles that of $-2x^4$ for large values of $|x|$

61. Could be the graph of a polynomial.
The zeros are -1, 1, and 2.
Since the graph has 2 turning points, the function must have degree at least 3

62. Could be the graph of a polynomial.
The zeros are -1 and 2.
Since the graph has 3 turning points, the function must have degree at least 4.

63. The graph has a discontinuity at $x=-1$; polynomials have "smooth" graphs, so this is <u>not</u> the graph of a polynomial.

64. The graph has a sharp corner at $x=0$. Polynomials have graphs w/o corners or "cusps", so this is <u>not</u> the graph of a polynomial

65. $f(x) = x(x-1)(x-2)$
$= x(x^2-3x+2)$
$f(x) = x^3-3x^2+2x$

66. $f(x) = x(x-1)^2(x-2)$
$= x(x^2-2x+1)(x-2)$
$= (x^3-2x^2+x)(x-2)$
$= x^4-2x^3+x^2-2x^3+4x^2-2x$
$f(x) = x^4-4x^3+5x^2-2x$

Note that $x=1$ must be a zero of multiplicity 2, 4, etc. because the graph only touches the x-axis here.

67. zeros -1 of mult. 1, 2 of mult 1, and 1 of mult 2.

$f(x) = a(x+1)(x-1)^2(x-2)$

We see $(0,1)$ is also on the graph, so that tells us

$1 = f(0) = a(0+1)(0-1)^2(0-2)$

$1 = a(1)(-1)^2(-2) = a(-2)$

so $a = -\frac{1}{2}$

$f(x) = -\frac{1}{2}(x+1)(x-1)^2(x-2)$

68. zeros $-1, 1, 2$ all of multiplicity 1.

$f(x) = a(x+1)(x-1)(x-2)$

we see $(0,-1)$ also on the graph, so

$-1 = f(0) = a(0+1)(0-1)(0-2) = 2a \quad a = -\frac{1}{2}$

$f(x) = -\frac{1}{2}(x+1)(x-1)(x-2)$

69. $f(x) = x^2(x-3)$

(1) The graph will look like that of x^3 for large values of $|x|$

(2) The x-intercepts will be 0 and 3

The y-intercept is $f(0) = 0^2(0-3) = 0$

(3) The polynomial f has zeros; 0 of multiplicity 2 and 3 of multiplicity 1.

The graph of f will touch the x-axis at $x=0$ and

cross the x-axis at $x=3$.

(4) f has degree 3, so the graph of f can have at most 2 turning points.

(5)

(6) $f(x) = x^2(x-3)$