Section 5.2 #13–24, 25–35 odd, 43, 47, 51

13. Domain: $\{x \mid x-3 \neq 0\} = \{x \mid x \neq 3\}$

14. Domain: $\{x \mid 3+x \neq 0\} = \{x \mid x \neq -3\}$

15. $\{x \mid (x-2)(x+4) \neq 0\} = \{x \mid x \neq 2 \text{ and } x \neq -4\}$

16. $\{x \mid (x+3)(4-x) \neq 0\} = \{x \mid x = -3 \text{ and } x \neq 4\}$

17. $\{x \mid 2x^2-5x-3 \neq 0\} = \{x \mid (2x+1)(x-3) \neq 0\} = \{x \mid x \neq \frac{-1}{2} \text{ and } x \neq 3\}$

18. $\{x \mid 3x^2+5x-2 \neq 0\} = \{x \mid (3x-1)(x+2) \neq 0\} = \{x \mid x \neq \frac{1}{3} \text{ and } x \neq -2\}$

19. $\{x \mid x^3-8 \neq 0\} = \{x \mid (x-2)(x^2+2x+4) \neq 0\} = \{x \mid x \neq 2\}$

20. $\{x \mid x^4-1 \neq 0\} = \{x \mid (x^2-1)(x^2+1) \neq 0\} = \{x \mid x \neq 1 \text{ and } x \neq 1\}$

21. $\{x \mid x^2+4 \neq 0\} = \mathbb{R}$

22. $\{x \mid x^4+1 \neq 0\} = \mathbb{R}$

23. $\{x \mid 4(x^2-9) \neq 0\} = \{x \mid 4(x+3)(x-3) \neq 0\} = \{x \mid x \neq 3 \text{ and } x \neq -3\}$

24. $\{x \mid 3(x^2+4x+4) \neq 0\} = \{x \mid 3(x+2)^2 \neq 0\} = \{x \mid x \neq -2\}$

25 (a) Domain $\{x \mid x \neq 2\}$ Range $\{y \mid y \neq 1\}$

(b) x-intercept: 0 y-intercept: 0

(c) Horizontal asymptote $y = 1$

(d) vertical asymptote $x = 2$

(e) no oblique asymptote

27 (a) Domain $\{x \mid x \neq 0\}$ Range $\mathbb{R}$

(b) x-intercepts: $-1, 1$ no y-intercept

(c) No horizontal asymptote

(d) No vertical asymptotes.

(e) Oblique asymptote: $y = 2x$

(The asymptote passes through $(0,0)$ and $(1,2)$, so it is a line of slope 2 and y-intercept 1).

29. (a) Domain $\{x \mid x \neq 2, x \neq -2\}$ Range $\{y \mid y \neq 1\}$

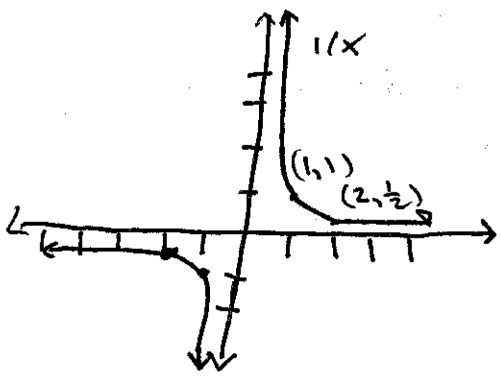
(b) x-intercept: 0 y-intercept: 0

(c) Horizontal asymptote: $y = 1$

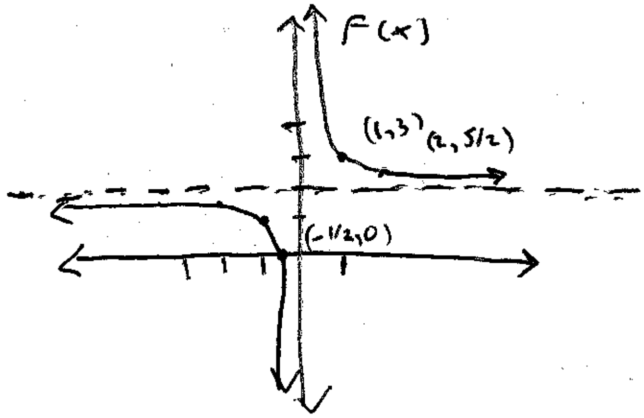
(d) vertical asymptotes: $x = -2$, $x = 2$

(e) No oblique asymptotes.

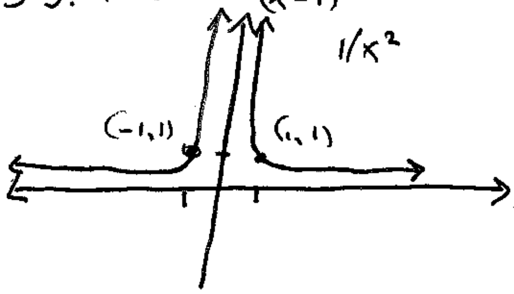
31. $f(x) = 2 + \frac{1}{x}$



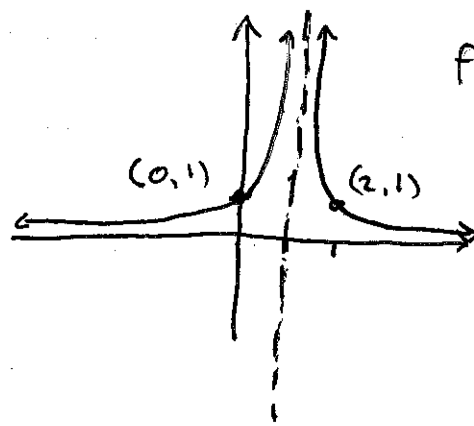
up 2 units



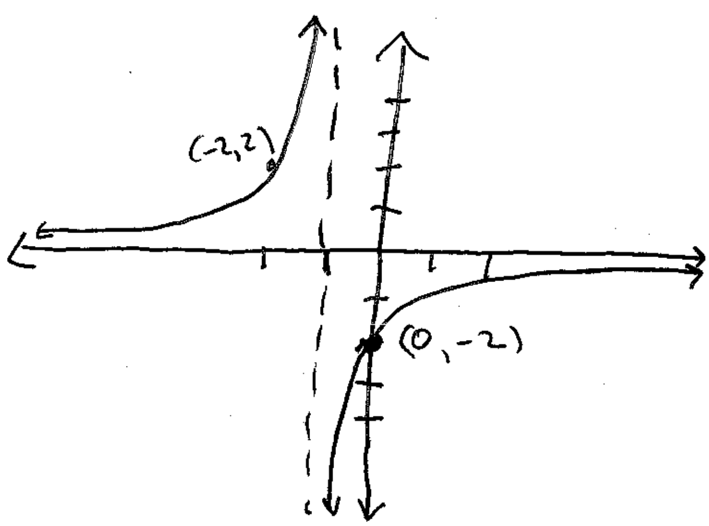
33. $f(x) = \frac{1}{(x-1)^2}$



right 1 unit



35. $\frac{-2}{x+1} = f(x)$ has the graph of $\frac{1}{x}$, shifted left 1 unit, stretched vertically by a factor of 2 and reflected over the x axis.



43. $R(x) = \dfrac{3x}{x+4}$

This is a <u>proper</u> rational function (numerator and denominator have same degree in this case), so the asymptotes occur at:

Horizontal: $y = \frac{3}{1} = 3$

Vertical: $x = -4$ (the only zero of the denominator)

There is <u>no</u> oblique asymptote.

47. $T(x) = \dfrac{x^3}{x^4 - 1}$

This is a <u>proper</u> rational function, with the degree of the denominator larger than the degree of the numerator. Thus the asymptotes are

Horizontal: $y = 0$

Vertical: $x = -1$ and $x = 1$

(The denominator factors as $(x-1)(x+1)(x^2+1)$)

There is <u>no</u> oblique asymptote

51. $R(x) = \dfrac{6x^2 + 7x - 5}{3x + 5}$; this is an improper rational function, use Long division

$$
\begin{array}{r|l}
 & 2x - 1 \\
\hline
3x+5 & 6x^2 + 7x - 5 \\
 & 6x^2 + 10x \\
\hline
 & -3x - 5 \\
 & -3x - 5 \\
\hline
 & 0
\end{array}
$$

So $R(x) = \dfrac{(3x+5)(2x-1)}{(3x+5)}$

and the graph will just be the line $y = 2x - 1$, with a hole at $x = \frac{-5}{3}$

(Technically, $y = 2x - 1$ is an oblique asymptote. The graph of a rational function may intersect an oblique asymptote.)

Section 5.5 # 11–43 EOO, 45–65 EOO

11. Remainder is $f(2) = 4(2^3) - 3(2^2) - 8(2) + 4 = 32 - 12 - 16 + 4 = 8$

15. Remainder is $f(-3) = 3(-3)^6 + 82(-3)^3 + 27 = 0$

19. Remainder is $f(\tfrac{1}{2}) = 2(\tfrac{1}{2})^4 - (\tfrac{1}{2})^3 + 2(\tfrac{1}{2}) - 1$

$$= 2 \cdot \frac{1}{16} - \frac{1}{8} + 1 - 1 = \frac{1}{8} - \frac{1}{8} + 1 - 1 = 0$$

23. f has degree 6, so it has at most 6 real zeros

27. f has degree 4, so it has at most 4 real zeros.

31. f has degree 6, so it has at most 6 real zeros

35. $f(x) = x^5 - 6x^2 + 9x - 3$

$p: \pm 1, \pm 3$ potential rational zeros: $\pm 1, \pm 3$

$q: \pm 1$

39. $f(x) = 6x^4 - x^2 + 9$

$p: \pm 1, \pm 3, \pm 9$ potential rational zeros: $\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{3}{2}, \pm \frac{9}{2}$

$q: \pm 1, \pm 2, \pm 3, \pm 6$

43. $f(x) = 6x^4 + 2x^3 - x^2 + 20$

$p: \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$q: \pm 1, \pm 2, \pm 3, \pm 6$

potential rational zeros:
$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20,$
$\pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{5}{3},$
$\pm \frac{10}{3}, \pm \frac{20}{3}, \pm \frac{1}{6}, \pm \frac{5}{6}$

45. $f(x) = x^3 + 2x^2 - 5x - 6$

potential rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

$f(1) = 1 + 2 - 5 - 6 = -8$ (not a zero)

$f(-1) = -1 + 2 + 5 - 6 = 0$ so $x+1$ is a factor of f

$$\begin{array}{r} x^2 + x - 6 \\ x+1 \enclose{longdiv}{x^3 + 2x^2 - 5x - 6} \\ \underline{x^3 + x^2 } \\ x^2 - 5x - 6 \\ \underline{x^2 + x } \\ -6x - 6 \\ \underline{-6x - 6} \\ 0 \end{array}$$

so $f(x) = (x+1)(x^2 + x - 6)$

$$\boxed{\begin{array}{c} f(x) = (x+1)(x-2)(x+3) \\ \text{with zeros } -1, -3, 2 \end{array}}$$

49. $f(x) = 2x^3 - 4x^2 - 10x + 20$

potential rational zeros: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm \frac{1}{2}, \pm \frac{5}{2}$

$p: \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$q: \pm 1, \pm 2$

$f(1) = 2 - 4 - 10 + 20 = 8$ (not a zero)

$f(-1) = -2 - 4 + 10 + 20 = 24$ (not a zero)

$f(2) = 2 \cdot 8 - 4 \cdot 4 - 10 \cdot 2 + 20 = 16 - 16 - 20 + 20 = 0$ so $x - 2$ is a factor

$$
\begin{array}{r|l}
 & 2x^2 - 10 \\
x - 2 & 2x^3 - 4x^2 - 10x + 20 \\
 & 2x^3 - 4x^2 \\
\hline
 & \quad -10x + 20 \\
 & \quad -10x + 20 \\
\hline
 & \qquad 0
\end{array}
$$

so $f(x) = (x-2)(2x^2 - 10)$

$f(x) = 2(x-2)(x^2 - 5)$

$\boxed{f(x) = 2(x-2)(x - \sqrt{5})(x + \sqrt{5})}$

with zeros $2, \sqrt{5}, -\sqrt{5}$

53. $f(x) = x^4 + x^3 - 3x^2 - x + 2$

potential rational zeros: $\pm 1, \pm 2$

$f(1) = 1 + 1 - 3 - 1 + 2 = 0$ so $x - 1$ is a factor

$$
\begin{array}{r|l}
 & x^3 + 2x^2 - x - 2 \\
x - 1 & x^4 + x^3 - 3x^2 - x + 2 \\
 & x^4 - x^3 \\
\hline
 & 2x^3 - 3x^2 - x + 2 \\
 & 2x^3 - 2x^2 \\
\hline
 & -x^2 - x + 2 \\
 & -x^2 + x \\
\hline
 & -2x + 2 \\
 & -2x + 2 \\
\hline
 & 0
\end{array}
$$

so $f(x) = (x-1)(x^3 + 2x^2 - x - 2)$

Let $g(x) = x^3 + 2x^2 - x - 2$

potential rational zeros: $\pm 1, \pm 2$

$g(1) = 1 + 2 - 1 - 2 = 0$ so $(x-1)$ is a factor

$$
\begin{array}{r|l}
 & x^2 + 3x + 2 \\
x - 1 & x^3 + 2x^2 - x - 2 \\
 & x^3 - x^2 \\
\hline
 & 3x^2 - x - 2 \\
 & 3x^2 - 3x \\
\hline
 & 2x - 2 \\
 & 2x - 2 \\
\hline
 & 0
\end{array}
$$

so $f(x) = (x-1)^2(x^2 + 3x + 2)$

$\boxed{f(x) = (x-1)^2(x+2)(x+1)}$

with zeros $-2, -1, 1$ of mult. 2

* 57. $x^4 - x^3 + 2x^2 - 4x - 8 = 0$ $\boxed{\text{has solutions } x = -1, 2}$

potential $\overset{\text{rational}}{\text{zeros}}$: $\pm 1, \pm 2, \pm 4, \pm 8$

Let $f(x) = x^4 - x^3 + 2x^2 - 4x - 8$

$f(1) = 1 - 1 + 2 - 4 - 8 = -10$ (not a zero)

$f(-1) = 1 + 1 + 2 + 4 - 8 = 0$ so $x+1$ a factor

$$
\begin{array}{r|l}
 & x^3 - 2x^2 + 4x - 8 \\
\hline
x+1 & x^4 - x^3 + 2x^2 - 4x - 8 \\
 & x^4 + x^3 \\
\hline
 & -2x^3 + 2x^2 - 4x - 8 \\
 & -2x^3 - 2x^2 \\
\hline
 & 4x^2 - 4x - 8 \\
 & 4x^2 + 4x \\
\hline
 & -8x - 8 \\
 & -8x - 8 \\
\hline
 & 0
\end{array}
$$

so let $g(x) = x^3 - 2x^2 + 4x - 8$

potential rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8$

$g(1) = 1 - 2 + 4 - 8 = -5$ (not a zero)

$g(-1) = -1 - 2 - 4 - 8 = 1$ (not a zero)

$g(2) = 8 - 8 + 8 - 8 = 0$ so $x-2$ is a factor

$$
\begin{array}{r|l}
 & x^2 + 4 \\
\hline
x-2 & x^3 - 2x^2 + 4x - 8 \\
 & x^3 - 2x^2 \\
\hline
 & 4x - 8 \\
 & 4x - 8 \\
\hline
 & 0
\end{array}
$$

so $g(x) = (x-2)(x^2+4)$ and x^2+4 is an irreducible quadratic, so we're done.

61. $3x^3 - x^2 - 15x + 5 = 0$

potential rational zeros: $\pm 1, \pm 5, \pm \frac{1}{3}, \pm 5$

Let $f(x) = 3x^3 - x^2 - 15x + 5$.

$f(1) = 3 - 1 - 15 + 5 = -8$ (not a zero)

$f(-1) = -3 - 1 + 15 + 5 = 16$ (not a zero)

$f(5) = 3 \cdot 5^3 - 5^2 - 15 \cdot 5 + 5 = 280$

$f(-5) =$

$f(\frac{1}{3}) = 3(\frac{1}{3})^3 - \frac{1}{9} - \frac{15}{3} + 5 = 0$ so $x - \frac{1}{3}$ is a factor.

$f(x) = 3(x - \frac{1}{3})(x^2 - 5) = 3(x - \frac{1}{3})(x + \sqrt{5})(x - \sqrt{5})$

so the solutions are $x = -\sqrt{5}, \sqrt{5}, \frac{1}{3}$.

65. $x^3 - \frac{2}{3}x^2 + \frac{8}{3}x + 1 = 0$

~~potential rational zeros: ±1~~

$\cancel{1^3 - \frac{2}{3} + \frac{8}{3} + 1 = 2 + \frac{6}{3} = \frac{12}{3}}$ ~~(not a zero)~~

$\cancel{-1 - \frac{2}{3}}$

We can only use the rational zeros theorem if all of the coefficients are integers.

$f(x) = 3x^3 - 2x^2 + 8x + 3 = 3\left(x^3 - \frac{2}{3}x^2 + \frac{8}{3}x + 1\right)$ has the same set of zeros, so we'll find zeros of $f(x)$.

Well f has potential rational zeros $\pm 1, \pm 3, \pm \frac{1}{3}$

$f(1) = 3 - 2 + 8 + 3 = 12$

$f(-1) = -3 - 2 - 8 + 3 = -10$

$f(3) = 3(27) - 2 \cdot 9 + 8 \cdot 3 + 3 = 81 - 18 + 24 + 3 = 108 - 18 = 90$

$f(-3) = -81 - 18 - 24 + 3 = -123 + 3 = -120$

$f\left(\frac{1}{3}\right) = 3\left(\frac{1}{27}\right) - 2\left(\frac{1}{9}\right) + \frac{8}{3} + 3 = \frac{1}{9} - \frac{2}{9} + \frac{8}{3} + \frac{9}{3} = \frac{17}{3} - \frac{1}{9} \neq 0$

$f\left(\frac{-1}{3}\right) = -3\left(\frac{1}{27}\right) - 2\left(\frac{1}{9}\right) - \frac{8}{3} + 3 = \frac{-1}{9} - \frac{2}{9} - \frac{8}{3} + \frac{9}{3} = \frac{-3}{9} + \frac{1}{3} = 0$

so $x + \frac{1}{3}$ is a factor of f. Use synthetic division to factor.

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & -2 & 8 & 3 \\ & & -1 & 1 & -3 \\ \hline & 3 & -3 & 9 & 0 \end{array}$$

so $f(x) = \left(x + \frac{1}{3}\right)\left(3x^2 - 3x + 9\right)$

$= 3\left(x + \frac{1}{3}\right)\left(x^2 - x + 3\right)$

But $g(x) = x^2 - x + 3$ has discriminant

$b^2 - 4ac = 1 - 4 \cdot 1 \cdot 3 = -11$,

so it has no real zeros. We're done

$\boxed{x = \frac{-1}{3} \text{ is the only real solution to } x^3 - \frac{2}{3}x^2 + \frac{8}{3}x + 1 = 0}$