Section 5.6 # 7, 11, 15, 19, 23, 25, 31, 33, 35

7. Degree 3; zeros 3, $4-i$

There must be a third zero, $\overline{4-i} = 4+i$

11. Degree 5; zeros: 1, i, $2i$

There must be two more zeros, $\overline{i} = -i$ and $\overline{2i} = -2i$.

15. Degree 6; zeros 2, $2+i$, $-3-i$, 0

There must be two more zeros, $\overline{2+i} = 2-i$ and $\overline{-3-i} = -3+i$.

19. Degree 5; zeros 2, $-i$, $1+i$

The other two zeros must be $\overline{-i} = i$ and $\overline{1+i} = 1-i$.

One such polynomial is

$$f(x) = (x-2)(x-i)(x+i)(x-(1+i))(x-(1-i))$$

$$f(x) = (x-2)(x^2+1)(x^2-2x+2)$$

$$f(x) = (x^3-2x^2+x-2)(x^2-2x+2)$$

$$= x^5-2x^4+2x^3-2x^4+4x^3-4x^2+x^3-2x^2+2x-2x^2+4x-4$$

$$\boxed{f(x) = x^5-4x^4+7x^3-8x^2+6x-4}$$

23. $f(x) = x^3-4x^2+4x-16$, zero: $2i$ another zero is $\overline{2i} = -2i$

So $(x-2i)(x+2i) = (x^2+4)$ must be a factor of $f(x)$.

Using ~~synthetic~~ long division:

$$\begin{array}{r|l} & x-4 \\ \hline x^2+4 & x^3-4x^2+4x-16 \\ & x^3+4x \\ \hline & -4x^2-16 \\ & -4x^2-16 \\ \hline & 0 \end{array}$$

So in factored form

$$f(x) = (x-2i)(x+2i)(x-4)$$

and the zeros are

$$\boxed{-2i,\ 2i,\ 4}$$

25. $f(x) = 2x^4+5x^3+5x^2+20x-12$; zero: $-2i$; Another zero is $\overline{-2i} = 2i$

so $(x-2i)(x+2i) = x^2+4$ must be a factor of $f(x)$.

Using ~~synthetic division~~ long division

$$\begin{array}{r|l} & 2x^2+5x-3 \\ \hline x^2+4 & 2x^4+5x^3+5x^2+20x-12 \\ & 2x^4+8x^2 \\ \hline & 5x^3-3x^2+20x-12 \\ & 5x^3+20x \\ \hline & -3x^2-12 \\ & -3x^2-12 \\ \hline & 0 \end{array}$$

so

$$f(x) = (2x^2+5x-3)(x^2+4)$$

$$= (2x-1)(x+3)(x^2+4)$$

$\boxed{\text{and the zeros are } -2i,\ 2i,\ -3,\ \tfrac{1}{2}}$

31. $f(x)=x^3-1$; $f(1)=0$ so $x-1$ is a factor.
use synthetic division

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

so $f(x)=(x-1)(x^2+x+1)$.

Let $g(x)=x^2+x+1$. Using the quadratic Formula:

$$x=\frac{-1\pm\sqrt{1-4\cdot1\cdot1}}{2}=\frac{-1}{2}+\frac{\sqrt{3}}{2}i \text{ or } \frac{-1}{2}-\frac{\sqrt{3}}{2}i$$

So the zeros are 1, $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$, $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ and in factored form

$$f(x)=(x-1)\left(x+\tfrac{1}{2}-\tfrac{\sqrt{3}}{2}i\right)\left(x+\tfrac{1}{2}+\tfrac{\sqrt{3}}{2}i\right)$$

33. $f(x)=x^3-8x^2+25x-26$

$f(2)=2^3-8\cdot4+25\cdot2-26=8-32+50-26=-24+24=0$

So $x-2$ is a factor of f. Using synthetic division:

$$\begin{array}{r|rrrr} 2 & 1 & -8 & 25 & -26 \\ & & 2 & -12 & 26 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

so $f(x)=(x-2)(x^2-6x+13)$.

Let $g(x)=x^2-6x+13$. Then using the quadratic formula again:

$$x=\frac{6\pm\sqrt{36-4\cdot1\cdot13}}{2}=\frac{6\pm\sqrt{-16}}{2}=\frac{6\pm4i}{2}=3+2i \text{ or } 3-2i$$

so f has zeros $2, 3+2i$ and $3-2i$, and in factored form

$$f(x)=(x-2)(x-3-2i)(x-3+2i)$$

35. $f(x)=x^4+5x^2+4$; $f(x)=(x^2+4)(x^2+1)$ by factoring.

x^2+4 has zeros $\pm2i$

x^2+1 has zeros $\pm i$

So $f(x)$ has zeros $-2i, -i, i, 2i$, and in factored form

$$\boxed{f(x)=(x-i)(x+i)(x-2i)(x+2i)}$$

Section 6.1 # 11–51 EOO, 26, 36, 44, 56

11. $f(x) = 2x \quad g(x) = 3x^2+1$

(a) $(f\circ g)(4) = f(g(4)) = f(3\cdot 4^2+1) = f(49) = 2\cdot 49 = 98$

(b) $(g\circ f)(2) = g(f(2)) = g(2\cdot 2) = g(4) = 3\cdot 4^2+1 = 49$

(c) $(f\circ f)(1) = f(f(1)) = f(2) = 4$

(d) $(g\circ g)(0) = g(g(0)) = g(1) = 3\cdot 1^2+1 = 4$

15. $f(x) = \sqrt{x} \quad g(x) = 2x$

(a) $(f\circ g)(4) = f(g(4)) = f(8) = \sqrt{8} = 2\sqrt{2}$

(b) $(g\circ f)(2) = g(f(2)) = g(\sqrt{2}) = 2\sqrt{2}$

(c) $(f\circ f)(1) = f(f(1)) = f(1) = 1$

(d) $(g\circ g)(0) = g(g(0)) = g(0) = 0$

19. $f(x) = \frac{3}{x+1} \quad g(x) = \sqrt[3]{x}$

(a) $(f\circ g)(4) = f(g(4)) = f(\sqrt[3]{4}) = \frac{3}{1+\sqrt[3]{4}}$

(b) $(g\circ f)(2) = g(f(2)) = g(1) = 1$

(c) $(f\circ f)(1) = f(f(1)) = f(\frac{3}{2}) = \frac{3}{3/2+1} = \frac{3}{5/2} = \frac{6}{5}$

(d) $(g\circ g)(0) = g(g(0)) = g(0) = 0$

23. $f(x) = \frac{x}{x-1} \quad g(x) = -\frac{4}{x}$

$(f\circ g)(x) = f(g(x)) = \frac{-4/x}{(-4/x)-1} \cdot \frac{x}{x} = \frac{-4}{-4-x} = \frac{4}{x+4}$

$\text{dom } f\circ g = \text{dom } g \cap \{x \mid g(x) \in \text{dom } f\}$

$= \{x \mid x \neq 0\} \cap \{x \mid g(x) \neq 1\} = \{x \mid x \neq 0\} \cap \{x \mid x \neq -4\}$

$= \{x \mid x \neq 0, x \neq -4\}$

27. $f(x) = x^2+1 \quad g(x) = \sqrt{x-1}$

$(f\circ g)(x) = f(g(x)) = (\sqrt{x-1})^2+1 = x-1+1 = x$

$\text{dom } f\circ g = \text{dom } g \cap \{x \mid g(x) \in \text{dom } f\}$

$= \{x \mid x \geq 1\} \cap \mathbb{R} = \{x \mid x \geq 1\}$

31. $f(x)=3x+1$; $g(x)=x^2$

(a) $(f\circ g)(x)=3x^2+1$; domain: all real numbers

(b) $(g\circ f)(x)=(3x+1)^2=9x^2+6x+1$; domain: all real numbers

(c) $(f\circ f)(x)=3(3x+1)+1=9x+4$; domain: all real numbers

(d) $(g\circ g)(x)=(x^2)^2=x^4$; domain: all real numbers.

35. $f(x)=\dfrac{3}{x-1}$; $g(x)=\dfrac{2}{x}$

(a) $(f\circ g)(x)=\dfrac{3}{2/x-1}=\dfrac{3x}{2-x}$; domain $\{x \mid x\neq 0, x\neq 2\}$

(b) $(g\circ f)(x)=\dfrac{2}{3/(x-1)}=\dfrac{2x-2}{3}$; domain $\{x \mid x\neq 1\}$

(c) $(f\circ f)(x)=\dfrac{3}{(3/x-1)-1}=\dfrac{3x-3}{4-x}=\dfrac{3-3x}{x-4}$; domain $\{x \mid x\neq 1, x\neq 4\}$

(d) $(g\circ g)(x)=\dfrac{2}{2/x}=x$; domain $\{x \mid x\neq 0\}$

39. $f(x)=\sqrt{x}$; $g(x)=2x+3$

(a) $(f\circ g)(x)=\sqrt{2x+3}$; domain $\{x \mid 2x+3\geq 0\}=\{x \mid x\geq -3/2\}$

(b) $(g\circ f)(x)=2\sqrt{x}+3$; domain $\{x \mid x\neq 0\}$

(c) $(f\circ f)(x)=\sqrt{\sqrt{x}}=\sqrt[4]{x}$; domain $\{x \mid x\geq 0\}$

(d) $(g\circ g)(x)=2(2x+3)+3=4x+9$; domain: all real numbers

43. $f(x)=\dfrac{x-5}{x+1}$; $g(x)=\dfrac{x+2}{x-3}$

(a) $(f\circ g)(x)=\dfrac{\frac{x+2}{x-3}-5}{\frac{x+2}{x-3}+1}\cdot\dfrac{x-3}{x-3}=\dfrac{x+2-5x+15}{x+2+x-3}=\dfrac{-4x+17}{2x-1}$

domain $\{x \mid x\neq 3\}\cap\{x \mid \frac{x+2}{x-3}\neq -1\}=\{x \mid x\neq 3, x\neq \frac{1}{2}\}$

(b) $(g\circ f)(x)=\dfrac{\frac{x-5}{x+1}+2}{\frac{x-5}{x+1}-3}\cdot\dfrac{x+1}{x+1}=\dfrac{x-5+2x+2}{x-5-3x-3}=\dfrac{3x-3}{-2x-8}=\dfrac{3-3x}{2x+8}$

domain $\{x \mid x\neq -1, x\neq -4\}$

(c) $(f\circ f)(x)=\dfrac{\frac{x-5}{x+1}-5}{\frac{x-5}{x+1}+1}\cdot\dfrac{x+1}{x+1}=\dfrac{x-5-5x-5}{x-5+x+1}=\dfrac{-4x-10}{2x-4}$ domain $\{x \mid x\neq -1, x\neq 2\}$

(d) $(g\circ g)(x)$

$=\dfrac{\frac{x+2}{x-3}+2}{\frac{x+2}{x-3}-3}\cdot\dfrac{x-3}{x-3}=\dfrac{x+2+2x-6}{x+2-3x+9}=\dfrac{3x-4}{-2x+11}=\dfrac{4-3x}{2x-11}$

domain $\{x \mid x\neq 3, x\neq \frac{11}{2}\}$

47. $f(x) = x^3 \quad g(x) = \sqrt[3]{x}$

$(f \circ g)(x) = f(g(x)) = (\sqrt[3]{x})^3 = x$

$(g \circ f)(x) = g(f(x)) = \sqrt[3]{x^3} = x$

51. $f(x) = ax + b \quad g(x) = \frac{1}{a}(x - b) \quad a \neq 0$

$(f \circ g)(x) = f(g(x)) = a\left(\frac{1}{a}(x-b)\right) + b = (x - b) + b = x$

$(g \circ f)(x) = g(f(x)) = \frac{1}{a}((ax+b) - b) = \frac{1}{a}(ax) = x$

26. $f(x) = x - 2 \quad g(x) = \sqrt{1-x}$

$(f \circ g)(x) = f(g(x)) = \sqrt{1-x} - 2$

$\text{dom}(f \circ g) = \text{dom}\, g \cap \{x \mid g(x) \in \text{dom}\, f\}$

$= \{x \mid x \leq 1\} \cap \mathbb{R} = \{x \mid x \leq 1\}$

36. $f(x) = \frac{1}{x+3} \quad g(x) = \frac{-2}{x}$

(a) $(f \circ g)(x) = \frac{1}{-\frac{2}{x} + 3} = \frac{x}{3x - 2}$; domain $\{x \mid x \neq 0, x \neq \frac{2}{3}\}$

(b) $(g \circ f)(x) = \frac{-2}{\frac{1}{x+3}} = -2x - 6$; domain $\{x \mid x \neq -3\}$

(c) $(f \circ f)(x) = \frac{1}{\frac{1}{x+3} + 3} = \frac{x+3}{3x+10}$; domain $\{x \mid x \neq -3, x \neq \frac{-10}{3}\}$

(d) $(g \circ g)(x) = \frac{-2}{-2/x} = x$; domain $\{x \mid x \neq 0\}$

44. $f(x) = \frac{2x-1}{x-2} \quad g(x) = \frac{x+4}{2x-5}$

(a) $(f \circ g)(x) = \frac{2\left(\frac{x+4}{2x-5}\right) - 1}{\left(\frac{x+4}{2x-5}\right) - 2} = \frac{2x + 8 - 2x + 5}{x + 4 - 4x + 10} = \frac{-13}{3x - 14}$ domain $\{x \mid x \neq \frac{5}{2}, x \neq \frac{14}{3}\}$

(b) $(g \circ f)(x) = \frac{\frac{2x-1}{x-2} + 4}{2\left(\frac{2x-1}{x-2}\right) - 5} = \frac{2x - 1 + 4x - 8}{4x - 2 - 5x + 10} = \frac{6x - 9}{-x + 8} = \frac{9 - 6x}{8 - x}$; domain $\{x \mid x \neq 2, x \neq 8\}$

(c) $(f \circ f)(x) = \frac{2\left(\frac{2x-1}{x-2}\right) - 1}{\frac{2x-1}{x-2} - 2} = \frac{2(2x-1) - x + 2}{2x - 1 - 2x + 4} = \frac{3x - 2}{3} = x - \frac{2}{3}$

domain $\{x \mid x \neq 2\}$

(d) $(g \circ g)(x) = \frac{\frac{x+4}{2x-5} + 4}{2\left(\frac{x+4}{2x-5}\right) - 5} = \frac{x + 4 + 8x - 20}{2x + 8 - 10x + 25} = \frac{9x - 16}{-8x + 33}$ domain $\{x \mid x \neq \frac{5}{2}, x \neq \frac{33}{8}\}$

56. $H(x)=\sqrt{1-x^2}$

If $f(x)=\sqrt{x}$

$g(x)=1-x^2$

Then $(f\circ g)(x)=f(g(x))=\sqrt{1-x^2}$

Alternately, let

$h(x)=\sqrt{1-x}$

$k(x)=x^2$

Then $(h\circ k)(x)=\sqrt{1-x^2}$ also.