Section 6.5 #13-77 EOO

13. $\log_3 3^{71} = 71$

17. $2^{\log_2 7} = 7$

21. $\log_6 18 - \log_6 3 = \log_6 \frac{18}{3} = \log_6 6 = 1$

25. $3^{\log_3 5 - \log_3 4} = 3^{\log_3 (5/4)} = 5/4$

29. $\ln 6 = \ln(3 \cdot 2) = \ln 3 + \ln 2 = b + a = a + b$

33. $\ln 8 = \ln(2^3) = 3\ln 2 = 3a$

37. $\log_5(25x) = \log_5 25 + \log_5 x = \log_5 5^2 + \log_5 x = 2 + \log_5 x$

41. $\ln(ex) = \ln e + \ln x = 1 + \ln x$

45. $\log_a(u^2v^3) = \log_a u^2 + \log_a v^3 = 2\log_a u + 3\log_a v$ (OK since $u, v > 0$)

49. $\log_2\left(\frac{x^3}{x-3}\right) = \log_2(x^3) - \log_2(x-3) = 3\log_2 x - \log_2(x-3)$

53. $\ln\left[\frac{x^2-x-2}{(x+4)^2}\right]^{1/3} = \ln\left[\frac{(x-2)(x+1)}{(x+4)^2}\right]^{1/3}$

$= \frac{1}{3}\left[\ln(x-2) + \ln(x+1) - 2\ln(x+4)\right]$

$= \frac{1}{3}\ln(x-2) + \frac{1}{3}\ln(x+1) - \frac{2}{3}\ln(x+4)$

57. $3\log_5 u + 4\log_5 v = \log_5 u^3 + \log_5 v^4$

$= \log_5 (u^3 v^4)$

61. $\log_4 (x^2-1) - 5\log_4 (x+1)$

$= \log_4 (x^2-1) - \log_4 (x+1)^5$

$= \log_4 \left(\frac{x^2-1}{(x+1)^5}\right) = \log_4 \left(\frac{(x+1)(x-1)}{(x+1)^5}\right) = \log_4 \left(\frac{x-1}{(x+1)^4}\right)$

65. $8\log_2(\sqrt{3x-2}) - \log_2 (\frac{4}{x}) + \log_2(4)$

$= \log_2 (3x-2)^{8/2} + \log_2 (\frac{4}{x})^{-1} + \log_2(4)$

$= \log_2 (3x-2)^4 + \log_2(\frac{x}{4}) + \log_2(4) = \log_2[x(3x-2)^4]$

69. $2\log_2(x+1) - \log_2(x+3) - \log_2(x-1)$

$= \log_2 (x+1)^2 - \log_2(x+3) - \log_2(x-1)$

$= \log_2 \frac{(x+1)^2}{(x+3)(x-1)} = \log_2\left(\frac{x^2+2x+1}{x^2+2x-3}\right)$

73. $\log_{1/3} 71 = (\log 71)/(\log \frac{1}{3}) = -\frac{\log 71}{\log 3} \approx -3.880$

77. $\log_\pi e = \frac{\ln e}{\ln \pi} = \frac{1}{\ln \pi} \approx 0.874$

Section 6.6 #5-57 EOO

5. $\log_4 x = 2 \Rightarrow x = 4^2 = \boxed{16}$

9. $\log_4(x+2) = \log_4 8 \Rightarrow x+2=8 \Rightarrow \boxed{x=6}$

13. $3\log_2 x = -\log_2 27 \Rightarrow \log_2 x^3 = \log_2 27^{-1}$ (used $\log_a M^r = r\log_a M$)

$\Rightarrow x^3 = 27^{-1} = (3^3)^{-1} = 3^{-3}$ (used 1-1)

$x = \sqrt[3]{3^{-3}} = 3^{-1} = \boxed{\frac{1}{3}}$

17. $\log x + \log(x+15) = 2$ (require $x > 0$)

$\log[(x)(x+15)] = 2$ (log of a product)

$x(x+15) = 10^2$ (definition of log)

$x^2 + 15x = 100 \Rightarrow x^2 + 15x - 100 = 0$

$\Rightarrow (x-5)(x+20) = 0$

$\Rightarrow \boxed{x=5}$ or $\cancel{x=-20}$ discard as $-20<0$

21. $\log_2(x+7) + \log_2(x+8) = 1$ (require $x > -7$)

$\log_2[(x+7)(x+8)] = 1$

$(x+7)(x+8) = 2^1$

$x^2+15x+56 = 2$

$x^2+15x+54 = 0$

$(x+6)(x+9) = 0 \Rightarrow \boxed{x=-6}$ or $\cancel{x=-9}$ discard as $-9<-7$

25. $\ln x + \ln(x+2) = 4$ (require $x>0$)

$\ln(x(x+2)) = 4$

$x(x+2) = e^4$

$x^2+2x-e^4 = 0$

$x = \dfrac{-2 \pm \sqrt{4+4e^4}}{2} = -1 \pm \sqrt{1+e^4}$ discard the negative solution

$\boxed{x = -1+\sqrt{1+e^4}} \approx 6.456$

29. $\log_{1/3}(x^2+x) - \log_{1/3}(x^2-x) = -1$

$\boxed{\text{see end of solutions}}$

33. $2^{x-5} = 8$

$= 2^3$

$x-5=3$

$\boxed{x=8}$

37. $8^{-x} = 1.2 \Rightarrow \ln 8^{-x} = \ln 1.2$

$\Rightarrow (-x)\ln 8 = \ln 1.2 \Rightarrow -x = \frac{\ln 1.2}{\ln 8} \Rightarrow \boxed{x = \frac{-\ln 1.2}{\ln 8}}$

≈ -0.088

41. $3^{1-2x} = 4^x \Rightarrow \ln 3^{1-2x} = \ln 4^x$

$\Rightarrow (1-2x)\ln 3 = x\ln 4$

$\Rightarrow \ln 3 - (2\ln 3)x = x\ln 4$

$\Rightarrow \ln 3 = x\ln 4 + (2\ln 3)x = x(\ln 4 + 2\ln 3)$

$\Rightarrow \boxed{x = \frac{\ln 3}{\ln 4 + 2\ln 3}} \approx 0.307$

45. $1.2^x = (0.5)^{-x}$

$\ln 1.2^x = \ln 0.5^{-x}$

$x\ln 1.2 = (-x)\ln 0.5$

$x\ln 1.2 + x\ln 0.5 = 0$

$x(\ln 1.2 + \ln 0.5) = 0 \Rightarrow \boxed{x=0}$

49. $2^{2x}+2^{x}-12=0$

$(2^{x})^2+2^{x}-12=0$

$(2^{x}-3)(2^{x}+4)=0$

$2^{x}-3=0$ or $\cancel{2^{x}+4=0}$ impossible

$2^{x}=3$

$\ln 2^{x}=\ln 3$

$x\ln 2=\ln 3 \Rightarrow \boxed{x=\frac{\ln 3}{\ln 2}} \approx 1.585$

53. $16^{x}+4^{x+1}-3=0$

$(4^{x})^2+4(4^{x})-3=0$

$4^{x}=\frac{-4\pm\sqrt{16-4\cdot1\cdot(-3)}}{2}=\frac{-4\pm\sqrt{28}}{2}=-2\pm\sqrt{7}$ discard negative solution

$\boxed{x=\log_4(-2+\sqrt{7})\approx -0.315}$

57. $3\cdot4^{x}+4\cdot2^{x}+8=0$

$3(2^{x})^2+4(2^{x})+8=0$

$2^{x}=\frac{-4\pm\sqrt{16-4\cdot3\cdot8}}{6}=\frac{-4\pm\sqrt{-80}}{6}$ $\boxed{\text{no real solution.}}$

29. $\log_{1/3}(x^2+x)-\log_{1/3}(x^2-x)=-1$

$\log_{1/3}\left(\frac{x^2+x}{x^2-x}\right)=-1$

$\frac{x^2+x}{x^2-x}=\left(\frac{1}{3}\right)^{-1}=3$

$x^2+x=3(x^2-x)=3x^2-3x$

$0=2x^2-4x=2x(x-2)$

$\cancel{x=0}$ or $\boxed{x=2}$ (discard $x=0$ as $0^2+0=0$, so the logs cannot be evaluated.

Section 6.7 #7-31 EOO

7. $A = P(1+\frac{r}{n})^{nt} = \$100 \cdot (1+\frac{0.04}{4})^{4 \cdot 2} \approx \$108.29$

11. $A = P(1+\frac{r}{n})^{nt} = \$600(1+\frac{0.05}{365})^{365 \cdot 3} \approx \$697.09$

15. $P = A(1+\frac{r}{n})^{-nt} = \$100(1+\frac{0.06}{12})^{-12 \cdot 2} \approx \$88.72$

19. $P = A(1+\frac{r}{n})^{-nt} = \$600(1+\frac{0.04}{4})^{-4 \cdot 2} = \$554.09$

23. $r_e = (1+\frac{r}{n})^n - 1 = (1+\frac{0.05}{4})^4 - 1 \approx 0.05095$

about 5.1%

27. For 6%, compounded quarterly:

$r_e = (1+\frac{0.06}{4})^4 - 1 \approx 0.06136$

For 6.25% compounded ~~monthly~~ annually

$r_e = (1+\frac{0.0625}{365})^{365} - 1 \approx \cancel{0.06432}\ 0.06449$

So 6.25% compounded ~~monthly~~ annually is the better deal

31. For a doubled investment, $A = 2P$ so set

$2\cancel{P} = \cancel{P}(1+\frac{r}{1})^{1 \cdot 3}$

$2 = (1+r)^3$

$\sqrt[3]{2} = 1+r \quad r = \sqrt[3]{2} - 1 \approx 0.25992$

26%