

Quiz #4

Name: Key

Please work alone, and keep your eyes on your own paper. Show *all* work (and justify any use of L'Hôpital's rule) to receive full credit.

1. (10 points) Find the indicated limit.

$$\lim_{x \rightarrow 0} \frac{\ln \cos(2x)}{7x^2}$$

has the form $\frac{0}{0}$

$$\stackrel{\textcircled{1}}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\cos(2x)} \cdot (-\sin 2x) \cdot 2}{14x}$$

still has form $\frac{0}{0}$

$$= \lim_{x \rightarrow 0} \frac{-\tan(2x)}{7x}$$

$$\stackrel{\textcircled{2}}{=} \lim_{x \rightarrow 0} \frac{-2 \sec^2(2x)}{7}$$

$$= \lim_{x \rightarrow 0} \frac{-2}{7 \cos^2(2x)} = \frac{-2}{7}$$

Rubric — Subtract 1 point if they drop $\lim_{x \rightarrow 0}$ early

- +2 Justify as $\frac{0}{0}$ first time (must say "has the form..." or similar)
- +2 Apply L'Hôpital 1 first time
- +2 Justify as $\frac{0}{0}$ second time
- +2 Apply L'Hôpital second time.
- +2 Final answer

If they write $\lim \dots = \frac{0}{0}$ do not give credit for the justification

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2. (10 points) Find the indicated limit.

$$\lim_{x \rightarrow 0} (x + e^{x/3})^{3/x}$$

has the indeterminate form 1^∞

Let $y = (x + e^{x/3})^{3/x}$
 $\ln y = \frac{3 \ln(x + e^{x/3})}{x}$

$\lim_{x \rightarrow 0} \ln y$ has the form $\frac{0}{0}$

② $\lim_{x \rightarrow 0} \frac{3}{x + e^{x/3}} \cdot (1 + \frac{1}{3} e^{x/3})$

$= \lim_{x \rightarrow 0} \frac{3 + e^{x/3}}{x + e^{x/3}} = \frac{3 + e^0}{0 + e^0} = \frac{3 + 1}{0 + 1} = \textcircled{4}$

Thus $\lim_{x \rightarrow 0} y = e^{\lim_{x \rightarrow 0} \ln y} = e^4$

Rubric subtract 1 point if they drop $\lim_{x \rightarrow 0}$ early

+2 Form 1^∞

+2 Consider $\ln y$ (can just write the new limit)

+1 \rightarrow Form $0/0$

+2 \rightarrow Apply L'Hopital

+1 $\rightarrow \lim_{x \rightarrow 0} \ln y = 4$

+2 Final answer: $\lim_{x \rightarrow 0} y = e^4$