MATH 166 Section C
June 21, 2013

Quiz #5

Name: Key

Please work alone, and keep your eyes on your own paper. Show *all* work to receive full credit.

1. (10 points) An explicit formula for a_n is given. Write the first five terms of $\{a_n\}$, determine whether the sequence converges or diverges, and, if it converges, find $\lim_{n\to\infty} a_n$.

$$a_n = (-1)^n \frac{n}{n+2}$$

$a_1 = (-1)\left(\frac{1}{1+2}\right) = \frac{-1}{3}$

$a_2 = (-1)^2\left(\frac{2}{2+2}\right) = \frac{1}{2}$

$a_3 = (-1)^3\left(\frac{3}{3+2}\right) = -\frac{3}{5}$

$a_4 = (-1)^4\left(\frac{4}{4+2}\right) = \frac{2}{3}$

$a_5 = (-1)^5\left(\frac{5}{5+2}\right) = \frac{-5}{7}$

$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{n}{n+2} = \lim_{n\to\infty} \frac{1}{1+2/n} = 1 \neq 0$

Since the limit is <u>not</u> 0, then

$\lim_{n\to\infty} a_n$ DNE, i.e. the sequence diverges.

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2. (10 points) Find the sum of the following series. HINT: It may help you to write out the first few terms of the series.

$$\sum_{k=0}^{\infty}\left[5\left(\frac{1}{4}\right)^k+\left(-\frac{1}{6}\right)^k\right]$$

$\sum_{k=0}^{\infty} 5\left(\frac{1}{4}\right)^k = 5+\frac{5}{4}+\frac{5}{16}+\dots$

is a convergent geometric series with $a=5$, $r=\frac{1}{4}$

and sum $\frac{a}{1-r}=\frac{5}{1-1/4}=\frac{5}{3/4}=\frac{20}{3}$

$\sum_{k=0}^{\infty}(-1/6)^k = 1-\frac{1}{6}+\frac{1}{36}-\frac{1}{216}+\dots$

is a geometric series with $a=1$, $r=-\frac{1}{6}$

and sum

$\frac{a}{1-r}=\frac{1}{1+1/6}=\frac{1}{7/6}=\frac{6}{7}$

since both are convergent

$\sum_{k=0}^{\infty}\left[5\left(\frac{1}{4}\right)^k+\left(\frac{-1}{6}\right)^k\right]=\sum_{k=0}^{\infty}5\left(\frac{1}{4}\right)^k+\sum_{k=0}^{\infty}\left(\frac{-1}{6}\right)^k$

$=\frac{20}{3}+\frac{6}{7}=\frac{140+18}{21}=\boxed{\frac{158}{21}}$