MATH 166 Section C
June 28, 2013

Quiz #6

Name: Key

Please work alone, and keep your eyes on your own paper. Show *all* work to receive full credit.

1. (10 points) Use the Integral Test to determine the convergence or divergence of the series.

$$\sum_{k=6}^{\infty} \frac{2}{7k+6}$$

The function $f(x) = \frac{2}{7x+6}$ is continuous, positive, nonincreasing on the interval $[6, \infty)$.

$$\int_6^\infty \frac{2}{7x+6}\,dx = \lim_{t\to\infty} 2\int_6^t \frac{1}{7x+6}\,dx$$

$$= \lim_{t\to\infty} \frac{2}{7} \ln|7x+6| \Big|_6^t$$

$$= \lim_{t\to\infty} \frac{2}{7}\left[\ln|7t+6| - \ln|48|\right]$$

$= \infty$, so the integral diverges

By the integral test, since $\sum_{k=6}^{\infty} \frac{2}{7k+6}$ and $\int_6^\infty \frac{2}{7x+6}\,dx$ converge or diverge together, and the integral diverges, we know that the series diverges

Rubric

+2 • $\int_6^\infty \frac{2}{7x+6}\,dx$ written

+2 • Replace integral w/ limit

+2 • Correct antiderivative

+2 • Integral diverges (enough to say $\int = \infty$)

+2 • Series diverges

Quiz #6

2. (10 points) Use the Limit Comparison Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+2n+3}$$

Let $a_n = \frac{n}{n^2+2n+3}$ and $b_n = \frac{1}{n}$

Then $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{n^2+2n+3} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{n^2}{n^2+2n+3}$

$$= \lim_{n\to\infty} \frac{1}{1+2/n+3/n^2} = 1$$

$0 < 1 < \infty$, so $\sum a_n$ and $\sum b_n$ converge or diverge together by LCT

$\sum b_n = \sum \frac{1}{n}$ is the harmonic series, which diverges,

so $\sum a_n = \sum \frac{n}{n^2+2n+3}$ diverges.

Rubric

+2 • Explicitly give their choice for comparison series

+2 • $\lim_{n\to\infty} \frac{a_n}{b_n}$ written

+2 • $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$ (correctly evaluated)

+1 • $0 < 1 < \infty$ so converge or diverge together

+1 • $\sum \frac{1}{n}$ diverges

+2 • original series diverges.